ITRC WS2: A METHOD STATEMENT FOR INFRASTRUCTURE NETWORK RISK ANALYSIS

12 October, 2012
Abstract
This document is part of an on-going process that will lead to the development of an infrastructure risk analysis for Great Britain. As part of the ITRC study of the “long term dynamics of interdependent infrastructure systems” this study deals the risk of failure of infrastructures in the present and future. This Work Stream 2 (WS2) analysis, out of five work streams, aims to build and apply analytical concepts, theoretical and simulation models and data tools to develop an integrated risk analysis framework.

The current document is a compilation of an initial method statement for the infrastructure risk analysis problem. The primary objectives satisfied in this report are:

1. Development of important definitions that will be used throughout the WS2 analysis.
2. Presentation of a mathematical formalisation of the risk analysis framework that deals with quantifying probabilistic hazards, infrastructure failure probabilities, damage evaluation, and economic loss estimation.
3. Building a framework that establishes a unified methodology for reliability analysis, damage assessment, loss estimation, and risk analysis.
4. Outline key issues and steps required for implementation of the risk analysis methodology.
# Table of Contents

Abstract ................................................................................................................. 2

1. Overview of framework .................................................................................. 4

2. Definitions ....................................................................................................... 8

3. Notation ............................................................................................................ 10

4. Reliability and risk analysis ............................................................................ 12
   4.1. Network reliability .................................................................................... 12
   4.2. Damage due to infrastructure failure ...................................................... 14
   4.3. Infrastructure network risk ...................................................................... 14

5. Components of risk analysis framework ....................................................... 17
   5.1. Hazard quantification .............................................................................. 17
      5.1.1. General quantification ..................................................................... 17
      5.1.2. ITRC relevant hazards ..................................................................... 18
   5.2. Infrastructure network modelling ............................................................. 20
      5.2.1. Generalised model .......................................................................... 20
      5.2.2. Network connectivity ..................................................................... 21
      5.2.3. Weighted connectivity .................................................................... 23
      5.2.4. ITRC network modelling .................................................................. 23
   5.3. Infrastructure failure ................................................................................. 24
      5.3.1. Assembling failed assets .................................................................. 24
      5.3.2. Infrastructure failure footprint .......................................................... 25
      5.3.3. Economic loss estimation .................................................................. 26

6. Implementation of ITRC risk analysis ............................................................ 31
   6.1. Executing the risk calculations ................................................................. 31
      6.1.1. Risk analysis using the EA flood hazard estimates ......................... 31
      6.1.2. Risk calculations using the spatial hazard estimates ....................... 32
   6.2. Workflows and Phases in ITRC ................................................................. 34
      6.2.1. Phase 1 ............................................................................................ 34
      6.2.2. Phase 2 ............................................................................................ 40
      6.2.3. Phase 3 ............................................................................................ 47
      6.2.4. Summary of tasks and responsibilities ............................................. 49
   6.3. Computation tools ..................................................................................... 56
1. **Overview of framework**

The overall aim of the ITRC WS2 is to develop a framework to analyse risk of failure of infrastructures. Five infrastructures, called National Infrastructures (NI), considered for risk analysis are energy, transport, water, waste and ICT\(^1\). These five are lifeline systems because they are important for the proper functioning of the United Kingdom (UK) economy, society and security.

While there are different external or internal shocks or events which put infrastructures at risk, we are interested in the risk of failures due to external climate events. Extreme climate events have the potential of causing widespread infrastructure disruptions and are thus referred to as hazard events. The climate variables such as rainfall, wind, temperature that result in the hazards such as flooding, subsidence, windstorms and heat waves put infrastructures at risk.

The focus upon climatic events is motivated by a number of pragmatic issues:

1. Other hazards, including security threats and accidental failures, are extensively dealt with in other research projects.
2. Climate risks are a particular concern of a number of our stakeholders (e.g. Defra, Environment Agency, Network Rail, JBA) and have received attention in a number of influential recent studies (including the UK Climate Change Risk Assessment, CCRA), but the risks to infrastructure are not well understood for the UK.\(^2\)
3. The ITRC team has particular expertise in risk analysis of natural hazards, and climate change in particular, building upon previous EPSRC funded projects, including the ARCADIA project.

Though our focus is primarily on climate hazards, aspects of the methodology will be transferrable to analysis of other hazards to infrastructure systems.

From a modelling perspective climate hazards are spatial (and temporal) extreme events. Uncertainty is ubiquitous in natural hazards and needs to be accounted for when quantifying the hazard\(^2\). ITRC WS2 is beginning by focussing upon flooding hazard which have been identified by the CCRA as the largest climate risk in the UK. At one level the flooding hazard is represented by flood maps that identify areas affected by river and sea flood events having a range of occurrence probabilities. Another level of model development results in spatially coherent quantification of flooding, which can be used to generate flood maps that quantify the flooding hazard in terms of the probability of exceeding a threshold flow (or depth) in a given time interval (e.g. weekly or annual or many years). Since rainfall events trigger floods, another approach for quantifying the flooding hazard is through estimating river or catchment flows from rainfall-runoff models. The flooding hazard in this case is also quantified in terms

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\(^1\) The UK Infrastructure Transitions Research Consortium (ITRC). Proposal for an inter-disciplinary research programme on: Long term dynamics of interdependent infrastructure systems.

\(^2\) Rougier, J. et al. 2010. SAPPUR: NERC scoping study on uncertainty and risks in natural hazards, summary and recommendations. Bristol Environmental Risk Research Centre (BRISK) University of Bristol, UK.
of the probability of exceeding a threshold flow (and later translated to depth) in a given time period, but it can also be updated over time due to present and future rainfall events. The annual exceedance probabilities calculated in the above modelling approaches are also interpreted in terms of the return period of the hazard events.

The ITRC analysis will in due course extend to analyse extreme heat (which may lead to rail buckling and plant overheating) and windstorms. In these instances the climate variables (temperature, wind speed) directly impact upon the infrastructures.

There is also interest in the ITRC to understand infrastructure risk of failures due to subsidence. A Natural Perils Directory (NPD) dataset provides an advanced database of pedo-climatic interpretations of soil-related geohazards at a UK scale. The underlying NPD incorporates assessments of geohazard subsidence vulnerability based on observed, empirical meteorological data and soil conditions across the UK. Another output of such analysis generates a flood extent map of the UK that is used in the ITRC analysis of flood vulnerability analysis.

The five infrastructures considered in ITRC are present throughout the UK, which means these are spatially distributed systems spread over large geographic areas. Further there are several components or assets within each infrastructure and across infrastructures that are connected physically or through flow of information. As such an overall spatial network topology can be identified to build a unified representation of infrastructures. The characterisation of the connectivity between the network elements is crucial in determining the risk propagation when networks fail. In the ITRC WS2 analysis there are three levels of modelling networks, which arise due to different interpretations of network connectivity. At the basic level infrastructure assets are represented as spatially distributed assets for which connectivity is binary based on rules that identify the reliance of assets on each other and their geographic proximity. Given further information of network assets and their linkages, connectivity is refined at the next level based on number of physical linkages between assets. Such networks are topological structures because they show how physical or logical flows take place across assets, but still do not show what those flow values are. The final level of network representation in the ITRC WS2 seeks to quantify the physical and logical flows identified in the topological network built in the previous level. Analysis at this level requires quantification of the capacities of network linkages and the ways in which flows redistribute when linkages are interrupted. Also of relevance is the role of storage in providing a buffer to interruptions in supply.

The ITRC WS2 risk analysis framework aims to provide answers to the following two queries: (i) What is the probability of NI failure in present and future climates? (ii) What are the potential consequences of major infrastructure failures for people and the economy? Given the levels of complexity of the hazard and infrastructure network models there are several factors that affect the failure probabilities and consequences. Network connectivity behaviour and properties play a significant role in determining the failure mechanisms and their implications. Hence, in the ITRC different answers to the risk calculation can be
obtained when we consider the different combinations of the climate and network models being developed. Bearing in mind WS2 risk analysis is divided into three Phases (1, 2 and 3) that address different aspects of the risk analysis problem.

1. Phase 1 is an infrastructure failure impact assessment framework that is devoid of any probabilistic calculations. It uses the spatial extent of the hazard (flood hazard maps and NDP flood extent maps) and the basic network models to identify infrastructure assets that are prone to failure because they lie within the floodplain or are dependent on other assets that failed due to direct flooding. Consequences to people (customers, labour force) and economy are then estimated from the damaged assets influence on surrounding populations and businesses.

2. Phase 2 is a probabilistic risk calculation framework that uses the different probabilistic climate models and the topological network models to calculate the failure probabilities of assets due to direct impact of the climate hazards. Network connectivity is then used to evaluate further indirect failure probabilities and mechanisms. Similar to Phase 1, the consequences to people and the economy are further estimated to produce an overall risk calculation. Also included in the Phase 2 calculations are future climate and infrastructure scenarios that create new risks of failure for the NI.

3. Phase 3 is a probabilistic risk calculation framework but distinguishes itself from the Phase 2 analysis because it includes network flow and storage properties. As such failure of assets can lead to redistribution of flows which leads to time-dependent risk calculations and resilience estimates for the NI. The probability of failure of the network depends upon the capacity of network links and the quantity of available storage. Phase 3 risk calculations also include future capacity and demand models for infrastructures along with climate models to evaluate evolving network risks and fluxes.

The overall risk calculation (outlined in this document and relevant mostly to Phase 2 and 3 calculations) for NI can be summarised in the following steps:

1. Identify and quantify the natural hazard in terms of its magnitude, spatial extent and probability density function.
2. Identify infrastructure assets and build networks based on connectivity rules.
3. Find the intersection of assets and hazards and calculate probabilities of failures of the assets given the hazard.
4. Calculate the overall network failure probabilities due to the combination of all the assets failed directly and indirectly due to the hazard.
5. Estimate the damage associated with an asset failure in terms of some metric of its influence on population and economy.
6. Find the asset risk by multiplying its probability of failure and the value of the damage incurred due to failure.
7. Find the overall network risk by considering the combination of risks due to failed assets.
In the Phase 2 calculation, Step 4 is determined from analysis of network connectivity, whilst in the Phase 3 calculation Step 4 incorporates consideration of flows, capacities and storage within the network.
2. Definitions

Below are some definitions which are relevant to the risk analysis methodology outlined in the rest of the document. Currently these definitions apply to the ITRC WS2 framework only, even though some terminology mentioned below is also used in other work streams. It is hoped that in the final ITRC statement all work stream definitions would be combined to form a common terminology relevant to the overall ITRC mandate. The definitions are given in the order in which they are first used in the method statement.

**Common cause failure** – The failure mechanism in which two or more systems (or sub-systems) are disrupted simultaneously due to some common effect. Such a failure occurs due to the geographical proximity of systems which expose them to common threats from the local environment.

**Cascading failure** – The failure mechanism in which a component failure in one system component causes failure of another component, thereby causing a disruption to the other systems (or sub-systems). Such failures can propagate further to cause widespread disruption effects.

**Fragility** – The probability of failure of a structure (or system) that is conditioned on the hazard. It is a measure of the susceptibility of the asset to being damaged by the hazard.

**Hazard function** – The spatial (and temporal) extreme value distribution of the hazard given in terms of the probability density function of the hazard measure.

**Hazard footprint** – The spatial (and temporal) effect of a hazard event quantified in terms of the spatial extent of the hazard event. The function which quantifies the hazard footprint is called the footprint function.

**Infrastructure** – A collection of technological and human organizational structures that come together to form interdependent networks that provide reliable flows of goods and services leading to economic productivity and human wellbeing.

**Asset** – The single technological or human organizational structure unit which is significant economically and strategically for establishing and maintaining infrastructure functionality. Few examples of assets include power plants, energy substations, water treatment plants, roads, data centres.

**Dependence** – A connection established between two infrastructure assets, where the condition of one asset is influenced by the other but is not reciprocated. Dependence establishes a unidirectional relationship between infrastructure assets.

**Interdependence** – The reciprocal dependence between two assets that establishes bidirectional connectivity between network elements. Interdependence is the general term that is used to include dependence. It can be said that dependence implies that the interdependence is unidirectional.
Network – The graphical interpretation of an infrastructure that establishes topology and relationships between assets. In a network assets are assigned to be arranged as nodes and edges based on their functionality.

Node – An infrastructure asset that acts as a source of production, consumption, or transformation of goods or services.

Edge – An infrastructure asset in the form of a physical or virtual entity that acts as a conduit for flow for technology, information or influence. An edge between two nodes represents a direct level of interdependence.

Graph – The mathematical representation of a network. A graph is represented as a 3-tuple consisting of sets of nodes, sets of edges and sets of functions that map edges onto node pairs. The graph representation of the network is considered to be static if the nodes, edges and mapping functions do not change with time. The graph can also be dynamic if either or all of the nodes, edges and mapping functions change with time.

Weighted network – A network is said to be weighted when there exist measures for the strength of the connectivity between node pairs. Weights can be interpreted in terms of the number of edges between node pairs or the amount of flow between them.

Adjacency matrix – The square matrix representation of the function that maps the edges between node pairs. Elements of the adjacency matrix take are value of zero when two nodes are not linked, or a non-zero value signifying the existence and weight of the connectivity between the node pairs.

Infrastructure failure footprint – The measure of influence of an infrastructure in terms of the area it affects when failed.

Loss operator – A measure of the loss in terms of number of ‘customers’ affected due to infrastructure failure.
3. Notation

The table below gives the list of the mathematical symbols (with explanations) introduced while developing the risk analysis framework. The list is arranged in the order in which the particular symbol is first used in the method statement.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Set of assets that collectively form the infrastructure</td>
</tr>
<tr>
<td>$r()$</td>
<td>A binary function that denotes whether an assets is failed or not failed</td>
</tr>
<tr>
<td>$\vec{r}$</td>
<td>The binary vector containing the states of all assets of the infrastructure</td>
</tr>
<tr>
<td>$h$</td>
<td>General notation for a hazard measured at a point in space</td>
</tr>
<tr>
<td>$\mathbb{H}$</td>
<td>General notation for a hazard measured over a topography</td>
</tr>
<tr>
<td>$L(r_i</td>
<td>h)$</td>
</tr>
<tr>
<td>$\mathbb{P}$</td>
<td>Notation for probability</td>
</tr>
<tr>
<td>$f(h)$</td>
<td>General expression for probability density function of a hazard at a point in space</td>
</tr>
<tr>
<td>$P(r_i)$</td>
<td>Unconditional probability of failure of an asset</td>
</tr>
<tr>
<td>$f(\mathbb{r})$</td>
<td>General expression for probability density function of a hazard over topography</td>
</tr>
<tr>
<td>$P(\mathbb{r})$</td>
<td>Unconditional probability of failure of the network</td>
</tr>
<tr>
<td>$D_d(r_i)$</td>
<td>Measure of direct damage to infrastructure asset $a_i$</td>
</tr>
<tr>
<td>$D_d(\mathbb{r})$</td>
<td>Measure of direct damage to infrastructure network</td>
</tr>
<tr>
<td>$D_{in}(r_i)$</td>
<td>Measure of indirect damage to infrastructure network</td>
</tr>
<tr>
<td>$R_d(r_i)$</td>
<td>Measure of direct failure risk to infrastructure asset $a_i$</td>
</tr>
<tr>
<td>$R(\mathbb{r})$</td>
<td>Measure of risk of failure of entire infrastructure network</td>
</tr>
<tr>
<td>$Y(x_t)$</td>
<td>Climate parameter measured in space and time</td>
</tr>
<tr>
<td>$Y_{tr}$</td>
<td>Threshold value for the climate parameter used for hazard quantification</td>
</tr>
<tr>
<td>$T$</td>
<td>A lengthy time-interval over which climate data is collected to measure hazard distribution</td>
</tr>
<tr>
<td>$\Delta A(x)$</td>
<td>Area around the location $x$ which shows the spatial extent of the hazard</td>
</tr>
<tr>
<td>$H(x)$</td>
<td>Hazard function that denotes the overall spatial extent of a hazard</td>
</tr>
<tr>
<td>$\theta_{ct}$</td>
<td>A critical threshold temperature that indicates heatwave hazard</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of national infrastructures</td>
</tr>
<tr>
<td>$l^k$</td>
<td>Network notation for $k^{th}$ infrastructure</td>
</tr>
<tr>
<td>$N^k$</td>
<td>Set of nodes in the $k^{th}$ infrastructure network</td>
</tr>
<tr>
<td>$E^{kk}$</td>
<td>Set of edges within the $k^{th}$ infrastructure network</td>
</tr>
<tr>
<td>$M^{kk}$</td>
<td>Mapping function between edge set and node set for $k^{th}$ infrastructure</td>
</tr>
<tr>
<td>$q_k$</td>
<td>Number of elements in the set $N^k$</td>
</tr>
<tr>
<td>$p_{kk}$</td>
<td>Number of elements in the set $E^{kk}$</td>
</tr>
<tr>
<td>$I$</td>
<td>Collection of all national infrastructures</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of all nodes across all national infrastructures</td>
</tr>
<tr>
<td>$E$</td>
<td>Set of all edges across all national infrastructures</td>
</tr>
<tr>
<td>$M$</td>
<td>Mapping function between edge set and node set for all national infrastructures</td>
</tr>
<tr>
<td>$\mid\mid$</td>
<td>Notation for number of elements in a set</td>
</tr>
<tr>
<td>$\subseteq$</td>
<td>Implies that a set of a subset of or equal to another set</td>
</tr>
<tr>
<td>$\setminus$</td>
<td>Notation for difference between two sets</td>
</tr>
<tr>
<td>$\sim$</td>
<td>Denotes the existence of physical interdependency between nodes</td>
</tr>
<tr>
<td>$d(.)$</td>
<td>Distance operator for calculating the geographic separation between nodes</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Spherical radius of influence of each asset on surrounding assets</td>
</tr>
<tr>
<td>$(.,.)_p$</td>
<td>Notation for physical interdependence between network nodes</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( (\cdot)_G )</td>
<td>Notation for geographic interdependence between network nodes</td>
</tr>
<tr>
<td>( (\cdot)_C )</td>
<td>Notation for cyber interdependence between network nodes</td>
</tr>
<tr>
<td>( (\cdot)_L )</td>
<td>Notation for logical interdependence between network nodes</td>
</tr>
<tr>
<td>( (\cdot) )</td>
<td>Notation for overall interdependence between network nodes</td>
</tr>
<tr>
<td>( w )</td>
<td>Weight given to measure interdependence between network nodes</td>
</tr>
<tr>
<td>( G(\mathbf{x}) )</td>
<td>Function that signifies the geographic location of an asset on the map</td>
</tr>
<tr>
<td>( A^{\text{dir}}_{\text{fail}} )</td>
<td>Set of failed asset nodes that lie within the flooded area</td>
</tr>
<tr>
<td>( A^{\text{indir}}_{\text{fail}} )</td>
<td>Set of asset nodes that fail due to interdependence</td>
</tr>
<tr>
<td>( \mathcal{S} )</td>
<td>Infrastructure failure footprint measure</td>
</tr>
<tr>
<td>( \mathcal{C} )</td>
<td>Set containing loss operator estimates for infrastructure assets</td>
</tr>
<tr>
<td>( \mathbf{q} )</td>
<td>Vector of economic outputs of infrastructure sectors</td>
</tr>
<tr>
<td>( \mathbf{A} )</td>
<td>Matrix of economic interdependencies between infrastructure sectors</td>
</tr>
<tr>
<td>( \mathbf{c} )</td>
<td>Vector of economic demands for infrastructure sector outputs</td>
</tr>
<tr>
<td>( p(h_k) )</td>
<td>Discrete approximation to the continuous hazard probability density function</td>
</tr>
<tr>
<td>( \nu(h_k) )</td>
<td>A importance sampling function</td>
</tr>
</tbody>
</table>
4. Reliability and risk analysis

4.1. Network reliability

Infrastructure is vulnerable to external shocks because they are capable of damaging whole or parts of its assets. Such damages can occur initially within a confined set of asset boundaries but can spread out further due to the connected nature of the infrastructure. To understand infrastructure failure we need to develop knowledge about the reliability of systems. For quantification purposes reliability is the measure of the probability of failure, which is studied at the assets level and then at the network level.

The kind of network failures we are interested in quantifying are called common cause failures and cascading failures. Common cause failures occur due to the spatial extent of the climate hazards, which affect several assets simultaneously. Under a cascading failure process the direct physical disruption of one infrastructure asset propagates to cause indirect disruption to other connected assets. Such disruptions can have further failure effects that travel across the network.

While calculating the reliability of an individual asset is straightforward, the network reliability is complex due to the mechanisms of iterative failure propagation between the assets. Initial failure of some infrastructure assets leads to further failures which results in an ordered mechanism of failure that spreads across the network. Overall network failure might occur only when certain sets of assets have failed due to their importance and connectivity in the network. Once we have defined the infrastructure network we can assess the failure mechanisms.

Consider the set \( A \) as the collection of all the infrastructure assets that have been assigned properties in the infrastructure. An element of the set \( A \), denoted as \( a_i \), signifies the existence of an asset at a location on the map and there are \( b \) number of assets in the network. Hence we define an infrastructure by the following mathematical notation

\[
A = \{\{a_i\}, |A| = b\}
\]

Associated with each asset is a function \( r_i \) that defines the state of asset \( a_i \), where \( r_i = 0 \) denotes \( a_i \) being in a ‘failed’ state and \( r_i = 1 \) denotes \( a_i \) being in a ‘non-failed’ state. Note that here we deal with \( r_i \) as a mapping to binary values \( r_i: r_i \rightarrow \{0,1\} \), but in Phase 3 \( r_i \) may be extended to map to real numbers on the range \((0,1)\) denoting partial functionality. For the entire network all the asset states can be collected into a binary vector \( \mathbf{r} = (r_1, r_2, ..., r_b) \), whose elements are either 0 or 1 describing which assets have failed and which have not failed.

Given an asset \( a_i \) its fragility is its conditional probability of failure with respect to a hazard \( h \) (at its location) and is quantified as

\[
L(r_i|h) = \mathbb{P}[r_i = 0|h]
\]

(2)
Given the probability density function, \( f(h) \) of the hazard (at a location in space) and the conditional probability of asset failure for the given hazard, the unconditional failure probability of an individual asset \( P(r_i) \) is calculated by integrating the fragility with the hazard function:

\[
P(r_i) = \int_L L(r_i|h)f(h)dh
\]  

(3)

For calculating network reliability we need to include all possible failure combinations of assets that result in network failure. In the most exhaustive case we will have to consider all the possible \( 2^b \) failure combinations\(^3\), but if the properties of the network are known then the number of scenarios can be narrowed down to a few. We assume that there are \( n \) failure combinations (or ‘system states’) that contribute to overall network failure. The vector \( r \) defined before represents just one of the possible \( n \) failure combinations. We define the vector \( r^j = (r_1^j, r_2^j, ..., r_b^j) \) to represent the \( j^{th} \) failure combination and the tensor \( R = \{r^1, r^2, ..., r^n\} \) as the collection of \( n \) failure combinations that contribute to overall network failure. The hazard which initiates the failure now has to be considered over the entire network and is represented by vector \( h \).

The probability that the network is in state \( r^j \) depends upon the joint states of the assets and the probability density function \( f(h) \) for the hazard over the entire network and is given as

\[
P(r^j) = \int_h \mathbb{P}[r_1^j|h \cap r_2^j|h \cap ... \cap r_b^j|h]f(h)dh
\]  

(4)

This is calculated as\(^4\)

\[
P(r^j) = \int_h \prod_{i=1}^b \max \left[ \max \left[ 0, 1 - r_i^j \right], r_i^j \right] L(r_i|h), r_i^j \left[ 1 - L(r_i|h) \right] f(h)dh
\]  

(5)

The overall network reliability is given as\(^5\)

\[
P(R) = \sum_{j=1}^n 1_j P(r^j)
\]  

(6)

where \( 1_j \) is the indicator function, which is equal to 1 if the \( j^{th} \) combination causes network failure and is 0 if it does not cause network failure. In essence the above formulation is a fault

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\(^3\) Each asset can have 2 states: failed or not failed. Hence for \( b \) assets there are \( 2 \times 2 \times ... 2 (b \text{ times}) = 2^b \) state combinations.

\(^4\) Asset \( r_i \) fails with conditional probability \( L(r_i|h) \) or does not fail with probability \( 1 - L(r_i|h) \). Only one of these can happen which is represented by the operation \( \max \left[ \max \left[ 0, 1 - r_i^j \right], r_i^j \right] L(r_i|h), r_i^j \left[ 1 - L(r_i|h) \right] \). For all \( b \) assets the overall probability is the product of individual probabilities.

\(^5\) Since any of the \( r^j \) mechanisms can lead to network failure the overall network failure probability is the sum of those mechanisms that are realised.
tree analysis quantification in which the sets $r^j$ are the events that make up the fault tree $\bar{r}$ that contains the entire collection of possible failure mechanisms\(^6\).

4.2. Damage due to infrastructure failure

We consider two types of damage associated with infrastructure failure:

1. The direct damage to the infrastructure itself. This may be quantified as the cost of reinstating an asset to the state it was in before it was damaged and is written as $D_d(r^j_i)$. It is reasonable to suppose that the total cost of reinstating the damaged assets in the failure mechanism set $r^j$ is given by adding individual asset damages.

$$D_d(r^j) = \sum_{i=1}^{b} \max[0,1-r^j_i] D(r^j_i) \quad (7)$$

2. The damage due to interruption of service to the customers (households, businesses, government) who depend upon the network. When the network is functioning as intended, customers incur some benefits from the network. When the network fails, they cease to incur those benefits and may incur some additional damage. The loss associated with network failure is the aggregate, over some appropriate timeframe, of the difference between the welfare that would have accrued had the network continued to function and the welfare in the case when the network fails. Thus, the loss for a network that does not fail is by definition equal to zero. In the case of indirect damage, we consider the consequences of failure (damage) to be determined by the overall functionality of the network: failure of an asset may cause no direct damage, but could disrupt the functioning of the network, causing interruptions to customers. In general we associate damages $D_{in}(r^j_i)$ with individual assets and $D_{in}(r^j)$ with network failure combination $r^j$. The overall damage associated with a given system failure state is computed as:

$$D_{in}(r^j) = \bigcup_{i=1}^{b} \max[0,1-r^j_i] D_{in}(r^j_i) \quad (8)$$

Here we have used a union instead of a summation because several assets might affect the same customers.

4.3. Infrastructure network risk

Infrastructure risk is calculated by considering the probabilities of asset failures and the damages that might be associated with such failures. Risk calculations proceed in a three step implementation: (i) finding the overall failure probabilities of the assets, (ii) finding the damages due to failed assets, and (iii) integrating the probabilities and damages to estimate overall risk.

---

For direct damage to asset $a_i$, the direct risk $R_d(r^i)$ can be calculated as the product of the magnitude of damage and the probability that damage occurs:

$$R_d(r^i) = D_d(r^i) \int_h L[r^i | h]f(h)dh \quad (9)$$

Note that in this formulation the severity of damage depends only upon the failure of the asset and is independent of the severity of the hazard.

The risk of failure of the network due to the failure mechanism $r^j$ is calculated by taking the product of the damage and the probability of being in that system state

$$R(r^j) = \left( D_d(r^j) + D_{in}(r^j) \right) P(r^j) = D(r^j)P(r^j) \quad (10)$$

As previously stated in the reliability calculations, $r^j$ is not the only mechanism of network failure but rather is an element of set $\bar{r}$ which includes all possible failure mechanisms that could lead to infrastructure failure. The overall network risk is estimated by considering all such possible mechanisms.

$$R(\bar{r}) = \sum_{j=1}^{n} 1_j R(r^j) \quad (11)$$

We are interested in calculating the risk due to the failure mechanism $r^j$ and further combine all the failure combinations to get the overall risk due to all possible failure combination given in $\bar{r}$. Since risk is the product of the damage and the failure probability it is

In order to implement the above risk calculation methodology we need to execute the following steps:

1. Identify the hazard and quantify it in terms of its magnitude (and spatial attributes) and probability density function.
2. Identify the infrastructure assets build the network based on connectivity rules link different assets.
3. Perform analysis on the topological network of assets and identify the set of network system failures due to direct hazard effects and also due to connectivity effects.
4. Find the probability of the failure of individual assets given the hazard. This gives the estimate of the asset fragility with respect to the hazard.
5. Calculate the probability of being in each failed system state.
6. Calculate the total probability of the network being in a failed state.
7. Estimate the damage associated with each asset failure and each failed system state.
8. Find the risk associated with each failed system state by multiplying its overall failure probability and the value of the damage incurred due to failure.
9. Find the overall network risk by considering the combination of all the failed set events.
Steps 1-6 constitute the network reliability analysis methodology, Step 7 is the damage assessment. Steps 1-9 constitute the entire risk methodology. The flowchart in Figure 1 summarises the steps outlined in the different frameworks above.

Figure 1: Framework for vulnerability and risk calculations required in the network failure analysis.
5. Components of risk analysis framework

Further explanation is provided now for more detailed quantification of the different components required for executing the risk analysis framework. Primarily we look at a generalised quantification for the three main components required in the analysis: (i) hazard quantification, (ii) infrastructure network modelling, and (iii) damage and loss estimation.

5.1. Hazard quantification

5.1.1. General quantification

Here we quantify the hazard in the most generalised manner without going into details of the climate models that generate the hazard. Our aim is to provide a basic understanding of the magnitude, spatial extent and probability distribution associated with climate parameters that are used for hazard quantification and further use in the risk framework.

Within the mandate of the ITRC WS2 hazard refers to an extreme climate event that is capable of causing damage to infrastructures. The different types of extreme climate types which are going to be studied in WS2 are flooding (fluvial, coastal, surface water), wind and heat. Also there is interest in analysing subsidence due to flooding, which is a geo-hazard. While each of the extreme climate types has different levels of complex models, in the end we are interested in quantifying the final outcome of these models in terms of a parameter that signifies the hazard. For example the flooding hazard is quantified in terms of the flood depth (or surface water flows), wind hazard is quantified in terms of the wind velocity, and heat hazard is quantified in terms of the temperature increase. In addition, there is also the spatial extent quantification that denotes the area of influence of the hazard.

Let $Y(x, t)$ denote the measure of the climate parameter that will be used for hazard quantification. The space $x$ denotes a point (3-dimensional) on a topography where $Y$ is recorded and the time $t$ denotes some relevant time-scale for which significant measures of $Y$ are available (example daily measures of rainfall or surface water flows or wind velocity or temperatures). For the same $x$ the values of $Y$ fluctuate due to uncertainty and randomness in climate properties. The values of $Y$ which are significant enough are classified as hazards. Generally a bigger time-scale $T$ over which several measures for $Y$ are available is used to quantify the hazard. Extremes climate situations that can cause damage are considered hazards, and as such the hazard parameter $h(x)$ is derived from the extreme values taken by $Y(x, t)$. Two possible ways of quantifying the hazard parameter are as follows:

1. A measure for the climate parameter that exceeds a given threshold value that is considered significant enough to cause a hazard

   $$ h(x) = Y(x, t) > Y_{tr}, \forall t \in (0, T] $$ (12)

2. The maximum value for the climate parameter measured over the longer time-scale

   $$ h(x) = \max_{t \in (0, T]} \{Y(x, t)\} $$ (13)
Once measures for the hazard parameter are obtained we can generate the hazard function $f_T(h(x))$ which represents the probability density function (or distribution) for the hazard measure. We are interested in quantifying the hazard function in terms of the probability of exceeding some threshold value. Hence the general formulation for the hazard function is given as

$$\int_{h_{tr}}^{\infty} f_T(h(x)) dh = \mathbb{P}[h \geq h_{tr} | x]$$  \hspace{1cm} (14)

A lower value for the exceedance probability signifies a large extreme event capable of causing large damage, while higher values denote an event that could be less damaging. The reciprocal of hazard function $\left(1/ \int_{h_{tr}}^{\infty} f_T(h(x)) dh \right)$, called the return period of the hazard event, is used many times to denote the severity of the hazard.

From the hazard function we are interested in obtaining the hazard footprint function\[^2\], which measures the spatial extent of the effect of the hazard measure. The hazard footprint function at a location is quantified in terms of the area ($\Delta A(x)$) of influence of the hazard that exists around the particular location $x$. Hence, the hazard footprint function $H(x)$ is represented as

$$H(x) = (1_{f > 0}) \Delta A(x)$$  \hspace{1cm} (15)

where the function operator $1_{f > 0}$ is an indicator function that is defined as

$$1_{f > 0} = \begin{cases} 1 & \text{if } f_T(h(x)) > 0 \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (16)

### 5.1.2. ITRC relevant hazards

For the ITRC WS2 risk due to flooding is the starting point of the analysis. There are three levels at which the flooding is being studied:

1. **Floodplain extraction from Environmental Agency (EA) data\[^7\]**: The final processed data is generated here as a flood map in which the definition of $h$ is more qualitative than quantitative. Instead of using flood depths the hazard is reported in terms of an event set given as $h = \{\text{Low, Medium, High, None}\}$, which makes $f_T(h(x))$ are discrete distribution defined as

$$f_T(h(x)) = \mathbb{P}\text{[Annual flooding by sea or river at location } x \text{ in any year]}$$  \hspace{1cm} (17)

Of greater interest from such data is the flood hazard footprint information $H(x)$ which is provided for low, medium and high probability flood events. Figure 2 explains the EA flooding data in greater detail.

\[^7\] The Environment Agency. 2012. Flood maps of UK.
2. **Spatial extreme models**[^8]: These are spatially coherent models that provide measures for extreme river or sea levels at multiple locations. The underlying model is a statistical conditional exceedance model, which is fitted to gauged data and describes the joint probability of extreme river flows or sea levels at multiple locations. \( f_T(h(x)) \) is defined as

\[
\int_{h_{tr}^1}^{\infty} f_T(h(x))dh = \mathbb{P}(h(x) > h_{tr}^1 | h(y) > h_{tr}^2)
\]

Where \( y \) is set as a baseline location for flow measure while \( h_{tr}^1 \) and \( h_{tr}^2 \) are two different threshold parameters for hazards. Typically the probabilistic measures are reported in terms of the return periods. Similar to EA flood extent maps \( H(x) \) measures can also be generated here by interpolation across gauge locations to obtain the comparable flood hazard footprint estimates.

3. **Flooding output from rainfall models**[^9][^10]: Rainfall models use daily rainfall data to generate continuous time-series spatially coherent rainfall estimates, which generate a climate ensemble of a few hundred events. Each rainfall event is generated on a 25km gridded area of the map, which can be disaggregated to 5km grid levels using weighting methods. Using the precipitation input data a grid-to-grid (G2G) rainfall flow routing and runoff-production model generates estimates of river flows and fluvial discharges into the sea. The analysis generates flow estimates (which might be further processed to get flood depths) on 1km (or 5 km or 25 km) grids on the entire map over river catchment areas. Such flow estimates are generated as a Monte Carlo sample of extreme events spanning many years. The hazard is quantified in terms of


the maximum annual flows and estimating their return periods. Hence $f_T(h(x))$ is estimated as

$$\int_{h_{tr}}^{\infty} f_T(h(x)) \, dh = \mathbb{P}[\text{Annual maximum flow } h(x) > h_{tr}]$$  \hspace{1cm} (19)

The G2G model can be used to provide flow estimates over a large area, which would lead to hazard footprint estimates. But it is better employed as a catchment model that gives flow estimates and return periods. Present and future rainfall climate scenario input makes such analysis useful for assessing future risks to infrastructures.

There is also interest in the ITRC to understand infrastructure risk of failures due to subsidence. A Natural Perils Directory (NPD)\(^{11}\) dataset provides an advanced database of pedo-climatic interpretations of soil-related geohazards at a UK scale. The underlying NPD incorporates assessments of geohazard subsidence vulnerability based on observed, empirical meteorological data and soil conditions across the UK. Another output of such analysis generates a flood extent map of the UK that is used in the ITRC analysis of flood vulnerability analysis. This provides measures for $H(x)$ as polygon flood area estimates derived from soil the type of soil, attributed in terms of whether it was laid down by river flood, lake sediment or marine deposit. There are no probabilistic dimensions to such estimates.

The rainfall models used in the flood hazard estimation can also be used to generate spatial-temporal heatwave models. Such models can provide heatwave metrics in terms of critical temperatures ($\theta_{cr}$) that are spatially correlated. Similar to flood hazard measures $f_T(h(x))$ for temperature hazard can be estimated as

$$\int_{\theta_{cr}}^{\infty} f_T(h(x)) \, dh = \mathbb{P}[h(x) > \theta_{cr}]$$  \hspace{1cm} (20)

Also interest lies in generating windstorm models capable spatial-temporal estimates of wind hazards. Wind velocity estimates in this case provide estimates for hazard quantification.

5.2. Infrastructure network modelling

5.2.1. Generalised model\(^{12}\)

Infrastructure is understood as a collection of assets that combine to provide functionality and economic wellbeing. Consider $m$ different national infrastructures of interest given by the set $k = \{1,2,\ldots,m\}$. Infrastructure $k$, denoted by $I^k$ is a network which is represented as a collection of nodes and edges. The graph representation of $I^k$ is given by the 3-tuple $\{N^k, E^{kk}, M^{kk}\}$, where $N^k$ is the set of nodes, $E^{kk}$ is the set of edges and $M^{kk}: E^{kk} \rightarrow$  

---


The edge set $E^{kk}$ therefore defines the interdependencies between the nodes of the infrastructure $k$. The three sets can be represented as

$$
N^k = \{n_1^k, n_2^k, \ldots, n_q^k\}, E^{kk} = \{e_1^{kk}, e_2^{kk}, \ldots, e_{p_{kk}}^{kk}\},
M^{kk} = \{e_1^{kk} \rightarrow (n_1^k, n_2^k), \ldots, e_{p_{kk}}^{kk} \rightarrow (n_i^k, n_j^k)\}
$$

where $q_k = |N^k|$ denotes the number of elements in the set $N^k$, $p_{kk} = |E^{kk}| = |M^{kk}|$ is the number of the elements of $E^{kk}$ and also $M^{kk}$, and $i \neq j \in \{1, 2, \ldots, q_k\}$.

The collection $I$ of the $m$ infrastructures includes internal as well as external interdependencies, which makes its representation complex. In simplest terms $I$ also is represented as a 3-tuple $\{N, E, M\}$, where $N$ is the set of nodes across all networks, $E$ is the set of edges across all networks and $M$ defines the mapping between edges and pair-wise nodes.

The example network illustrated in Figure 3 shows the nodes and edges within each infrastructure and also depicts the cross infrastructure connectivity. For this network the set structures defined above are explain below.

$$I^1: \left\{ \begin{array}{l}
N^1 = \{n_1^1, n_2^1, n_3^1\}, E^{11} = \{e_1^{11}, e_2^{11}, e_3^{11}\} \\
M^{11} = \{e_1^{11} \rightarrow (n_1^1, n_2^1), e_2^{11} \rightarrow (n_2^1, n_3^1), e_3^{11} \rightarrow (n_3^1, n_1^1)\} \\
\end{array} \right\}$$

$$I^2: \{N^2 = \{n_1^2, n_2^2\}, E^{22} = \{e_1^{22}\}, M^{22} = \{e_1^{22} \rightarrow (n_2^2, n_2^2)\} \}$$

$$I: \left\{ \begin{array}{l}
N = \{n_1^1, n_2^1, n_3^1, n_1^2, n_2^2\}, E = \{e_1^{11}, e_2^{11}, e_3^{11}, e_1^{12}, e_2^{12}, e_1^{22}\} \\
M = \{e_1^{11} \rightarrow (n_1^1, n_2^1), e_2^{11} \rightarrow (n_2^1, n_3^1), e_3^{11} \rightarrow (n_3^1, n_1^1), e_1^{12} \rightarrow (n_1^2, n_1^2) \\
e_2^{12} \rightarrow (n_2^1, n_2^1), e_1^{22} \rightarrow (n_2^2, n_2^2) \} \\
\end{array} \right\}$$

Figure 3: Example network showing connectivity within and across two different types of infrastructures.

### 5.2.2. Network connectivity

The existence of the mapping function is established based on rules of connectivity between the different infrastructure assets. As present we are interested in understanding the different
types of interdependence that exist due to existence of services, proximity and other effects between important assets. Given two nodes \( n_j \) and \( n_k \) in the complete network, the following interdependencies are mapped and quantified to understand the connectivity, which will further help in understanding failure propagation.

1. **Physical** – The existence of physical flows of services between infrastructure assets implies that they are physically interdependent. Such interdependence manifests due to the usage of output of one asset by another. As such failure in one infrastructure is directly transferrable towards failure in its physically interdependent infrastructure. The notation \(( n_j, n_k )_p\) is used to show the physical interdependency between the node pair, which also means that the failure in one node results in loss of physical services across the two, which translates to a physical interdependence failure.

2. **Geographical** – Spatial proximity between infrastructures results in a geographic interdependency due to common affects from the local environmental. The distance between the two nodes, given by \( d(n_j, n_k) \), is required to establish the geographical interdependence for failure. If \( \varepsilon \) denotes a radius of geographic influence of \( n_j \) and \( n_k \) lies within this distance then geographic interdependence exists between the two nodes. This is denoted as \( d(n_j, n_k) \leq \varepsilon \). Geographical interdependence is denoted as \(( n_j, n_k )_g\).

3. **Cyber** – When the interdependence between two infrastructure assets exists due to the flow of information across the two then it is called cyber interdependence. Denoted by \(( n_j, n_k )_c\) generally show the relation of the information technology infrastructure with other infrastructures. Hence at least one of the nodes in the relationship would be a cyber-infrastructure asset.

4. **Logical** – Interdependencies not represented through physical, geographic or logical connectivity are classified as logical interdependencies. In our analysis we can classify economic interdependencies as logical in nature. These are denoted as \(( n_j, n_k )_l\).

Based on the definitions and existence of different interdependencies the mapping between nodes in the infrastructure network is binary in nature and can be given by the rule

\[
(n_j, n_k) = \begin{cases} 
1 & \text{if } (n_j, n_k)_p \neq 0 \text{ or } (n_j, n_k)_g \neq 0 \text{ or } (n_j, n_k)_c \neq 0 \text{ or } (n_j, n_k)_l \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]  

(25)

This is also interpreted as assigning a value to the edge that shows the connectivity between the two nodes. If \( e_{jk} \) is the edge that will connect the two nodes then the value assigned to it can be given by

\[
e_{jk} = \begin{cases} 
1 & \text{if } (n_j, n_k)_p \neq 0 \text{ or } (n_j, n_k)_g \neq 0 \text{ or } (n_j, n_k)_c \neq 0 \text{ or } (n_j, n_k)_l \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

From the formulation developed above the network shown in Figure 3, which is considered to be a physically interdependent network can be represented with the interdependency adjacency matrix given in Table 1.

**Table 1: Interdependency mapping for the network in Figure 1**

<table>
<thead>
<tr>
<th></th>
<th>$n_1^1$</th>
<th>$n_2^1$</th>
<th>$n_3^1$</th>
<th>$n_1^2$</th>
<th>$n_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1^1$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$n_2^1$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$n_3^1$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n_1^2$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$n_2^2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

5.2.3. Weighted connectivity

While Table 1 shows the existence of connectivity between node pairs in terms of binary quantification, we are further interested in developing estimates for the strength or degree of the connectivity between them. Hence we need to assign values to the different interdependency mechanisms that reflect some strength of the connectivity. By assigning such values we are creating weighted networks that can be used in failure assessment. Instead of having a binary adjacency matrix we now have different values assigned to non-zero entries in the mapping. The Table 1 scheme for the example network would now be replaced by the representation in Table 2, in which the weighted mapping is given by the notation $w_{jk}^{ik} = (n_j^i, n_k^j)$.

**Table 2: Adjacency matrix for weighted network connectivity**

<table>
<thead>
<tr>
<th></th>
<th>$n_1^1$</th>
<th>$n_2^1$</th>
<th>$n_3^1$</th>
<th>$n_1^2$</th>
<th>$n_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1^1$</td>
<td>0</td>
<td>$w_{12}^{11}$</td>
<td>$w_{13}^{11}$</td>
<td>$w_{12}^{12}$</td>
<td>0</td>
</tr>
<tr>
<td>$n_2^1$</td>
<td>$w_{21}^{21}$</td>
<td>0</td>
<td>$w_{23}^{21}$</td>
<td>0</td>
<td>$w_{22}^{22}$</td>
</tr>
<tr>
<td>$n_3^1$</td>
<td>$w_{31}^{31}$</td>
<td>$w_{32}^{31}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n_1^2$</td>
<td>$w_{11}^{21}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$w_{12}^{22}$</td>
</tr>
<tr>
<td>$n_2^2$</td>
<td>0</td>
<td>$w_{21}^{21}$</td>
<td>0</td>
<td>$w_{22}^{22}$</td>
<td>0</td>
</tr>
</tbody>
</table>

5.2.4. ITRC network modelling

In the ITRC WS2 analysis the network modelling is challenging because we require information for the mapping function, which then generates the edge set. Hence, networks are developed at three levels or stages:

1. **Phase 1**: The network nodes are understood to be spatially distributed assets at different locations of the map. The mapping between node assets is binary measure of the existence of connectivity, based on the understanding that two nodes exchange resources and are geographically close to each other.

2. **Phase 2**: The network topology is better understood in terms of actual connectivity between nodes, which is established by finding the number of edges joining different nodes from available data showing asset location on the map. There are also some
connectivity measures that are not physically realised but exist due to non-physical effects between two nodes.

3. **Phase 3**: The topological network is refined by identifying the attributes of flow, capacity and fluxes for the network nodes and edges. Hence, with each node and edge we assign performance parameters that refine the connectivity measures.

### 5.3. Infrastructure failure

#### 5.3.1. Assembling failed assets
Failure is measured in terms of the number of infrastructure assets that are ‘damaged’ due to external climate hazard. An asset is said to be damaged when it is not able to perform its full function either to the physical breakdown of its components or due to loss of inputs required to generate its output. Failure of the infrastructures in initiated due to climate hazards that are distributed spatially. The hazard footprint \( H(x) \) is a polygon area showing the area extent on the map. An infrastructure asset is considered to be at risk of failure if it lies within the area polygon generated due to the hazard.

A defined previously, the set \( A \) is used to denote the collection of all the infrastructure assets that have been assigned as nodes and edges. An element of the set \( A \), denoted as \( a_i \), signifies the existence of an asset at a location on the map and there are a number of assets in the network. If for any infrastructure asset \( a_i \) the function \( G(a_i, x) \) denotes its coordinates on the map, then the set of assets failed directly due to the hazard is contained within the set \( A_{\text{fail}}^{\text{dir}} \), whose elements are defined as

\[
a_i^{\text{dir}} \equiv H(x) \cap G(a_i, x) = \begin{cases} a_i & \text{if asset lies within hazard area and is damaged} \\ \emptyset & \text{otherwise} \end{cases}
\]  

(27)

The failure is not restricted to only those assets that lie within its boundaries, because there are interdependent impacts that propagate beyond the extent of the hazard footprint. Once the directly failed assets have been identified from the intersection with the hazard, the indirectly failed assets can be identified from the interdependency rules defined before. An asset \( a_i \) belongs to the set of indirectly failed assets, \( A_{\text{fail}}^{\text{indir}} \), whose elements are given as

\[
a_i^{\text{indir}} = \begin{cases} a_i & \text{if } a_i \text{ is connected to } a_i^{\text{dir}} \text{ and fails} \\ \emptyset & \text{otherwise} \end{cases}
\]

(28)

The algorithm in Table 3 summarises the process of generating the set of directly and indirectly failed infrastructure assets.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Identify the polygon area extent of the hazard ( H(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2</td>
<td>Choose an infrastructure asset ( a_i ) &lt;br&gt; If ( a_i ) lies within the hazard polygon and fails include in in the set ( A_{\text{fail}}^{\text{dir}} ) &lt;br&gt; Else &lt;br&gt; Repeat the process with another asset till there are not more assets</td>
</tr>
<tr>
<td>Step 3</td>
<td>Choose a failed asset ( a_i^{\text{dir}} ) from the set ( A_{\text{fail}}^{\text{dir}} )</td>
</tr>
</tbody>
</table>
Find the asset $a_i$ that is connected to $a_i^{dir}$
If $a_i$ fails due to connectivity to $a_i^{dir}$ include it in the set $A_{fail}^{indir}$
else
Do the same for all other possible assets
End

The complete set of failed assets is the collection of directly and indirectly failed assets. Hence if $\bar{A}$ denotes the set of failed assets then $\bar{A} = A_{fail}^{dir} \cup A_{fail}^{indir}$.

### 5.3.2. Infrastructure failure footprint

Damage is evaluated in terms of the areas affected due to failed assets and customers within those areas affected due to the unavailability of services that were being provided by the failed infrastructure assets. Similar to a hazard footprint, an infrastructure failure footprint measures the area on the map affected by the collection of failed assets. Also for every asset a loss operator measures the number of customers affected due to its failure. When we have connected infrastructure assets the loss operators for directly and indirectly failed assets provide a complete picture of the overall customer loss due to infrastructure failures.

For any infrastructure asset $\bar{a}_i$ belonging to the failure set $\bar{A}$ we assume that the polygon area affected by its failure is denoted by $\bar{S}_i$. The overall infrastructure failure footprint is collection of all individual asset footprints and is given as

$$\bar{S} = \bigcup_{\forall \bar{a}_i} \bar{S}_i$$  \hspace{1cm} (29)

In cases when each individual asset affects a unique area the above expression is conveniently reduced to

$$\bar{S} = \sum_{\forall \bar{a}_i} \bar{S}_i$$  \hspace{1cm} (30)

For any given infrastructure asset $a_i$ let $\bar{C}_i$ denote the loss operator measured in terms of the number of customers that are using the resources it generates. To quantify $\bar{C}_i$ we need information on the area served by the asset and the customer count within the area. Hence for the asset $a_i$, $\bar{C}_i$ is a function of $S_i$ and is estimated as

$$\bar{C}_i = \bar{C}_i(a_i, S_i) = \text{No. of customers within the area } S_i$$  \hspace{1cm} (31)

Assuming the each individual asset serves a unique area, the overall damage set for failed assets can be used to construct a set of number of affected customers $\bar{C}$ given as

$$\bar{C}(\bar{A}) = \{ \bar{C}_1(\bar{a}_1, \bar{S}_1), \bar{C}_2(\bar{a}_2, \bar{S}_2), \ldots, \bar{C}_i(\bar{a}_i, \bar{S}_i), \ldots \}$$  \hspace{1cm} (32)

For a topological network data the infrastructure failure footprint and loss operator estimates are governed by the availability of customer data, which determined how accurate these estimates are. Currently in the ITRC such analysis is restricted at the postal code level based district boundaries into which UK is divided as shown in Figure 4. Further investigation into data shows that it is possible to further disaggregate estimates at the ward level. This would
provide more accurate ways of dividing the areas of influences and specifying the infrastructure failure footprint.

Figure 4: The district level map of the UK for which customer data can be obtained. Further disaggregated customer data is possibly available at the ward level.

5.3.3. Economic loss estimation

The economic loss estimation method outlined below is a static impact estimation method. It does not include temporal analysis of recovery and duration of losses. Such analysis is to be further developed in the ITRC analysis when we have estimates of network fluxes and flow redistributions.

The economic impacts of the failures of the selected infrastructures need to be extended beyond those assets which have been physically affected. Due to the interdependent nature of infrastructures there are indirect consequences of disruptions felt across multiple assets. While we are evaluating physical damage effects for the national infrastructures under consideration, economic impacts can be extended beyond this set due to the availability of data and the importance of considering a wider economic cascade effect that represents the severity of the damage.

During hazard events such as floods, we recognise that the flood can cause direct damages to households and businesses. In ITRC our interest is only in the infrastructure damages. We recognise three levels of damage due to infrastructure failure:

We recognise three categories of losses associated with infrastructure failure:

1. **Direct damages**: These refer to the direct damages to the infrastructure assets themselves costs incurred due to loss of capital, machinery.

2. **Losses to customers**: These are losses suffered by businesses that are dependent of the affected infrastructures. They include physical damage incurred due to failure of the infrastructure (e.g. due to failure of refrigeration), business interruption e.g. due to increased journey times and machinery down-time, and additional costs incurred associated with using back-up supplies.
3. **Indirect effects on the economy**: Such losses are generated due to multiplier affects in the economy, arising due to the forward linkages (propagated due to supply losses) and backward linkages (propagated due to demand losses).

While the physical effect of infrastructure failure can be evaluated at the individual asset level, the losses to customers depend on functionality at the network level. The computation of losses to customers requires estimation of the numbers of customers using each network and the scale of losses they may incur due to network failure. The only feasible way of estimating the indirect impacts upon the economy is *macroeconomic* in nature, is of interest to government entities that regulate the overall economic performance of the public sectors.

The key players in the macroeconomic analysis are comprised of industries, households and government:

1. **Industries**: These include the infrastructures which are affected by the hazard and businesses which depend on them. They provide goods and services for intermediary consumption of other businesses, final consumption of households and government.
2. **Households**: The role of the households is the demand goods and services from industries and supply labour to help industries produce these goods and services.
3. **Government**: Government purchases industry outputs thereby contributing to demand for goods and services and also provides facilities and tax incentives to help in the output supply.

The macroeconomic loss estimation is driven by the following direct effects:

1. **Infrastructures damages**: Infrastructures supply goods and services in the economy. Hence damage to infrastructures disrupts the flow of supply causing economic imbalance. In estimating the direct supply losses the set damaged assets $\mathcal{A}$ provides an estimate of the capital flow that is disrupted.
2. **Labour losses**: Infrastructures are dependent upon labour to produce their outputs. Due to infrastructure failure there is a loss of labour due to various factors such as inability to get to work, loss of life or psychological issues. Labour losses affect the direct supply of industry outputs. In estimating the supply losses due to labour losses we need to have estimates of the number transport assets disrupted and the number of households affected by those disruptions.
3. **Customers affected**: Industry outputs are used by customers to satisfy their final demands. Hence customers affected due to infrastructure failure results in losses of demand for the industry outputs. To measure the demand losses the loss operator $\mathcal{C}$ is used to provide the estimates for customer demand losses.
4. **Export and import losses**: Exports signify the final demands for industry outputs outside the regional domain of the industries, households and government. Imports signify the supply needed from outside to produce output but can be adjusted into final demand as a negative demand for industry outputs. Hence export-import losses are considered to be demand losses.
The economic input-output (I-O)\textsuperscript{14} modelling framework is used here to study the macroeconomic direct and indirect economic consequences of physical disruption effects to infrastructures. I-O framework consists of the following components:

1. Economic output of an infrastructures given as $q_i$.  
2. Interdependency matrix $A = [a_{ij}]$ showing the amount of economic output of infrastructure $i$ used by infrastructure $j$ for producing a single unit of its economic output.  
3. The household demands for the infrastructure output given as $c_{i,hh}$.  
4. The government sales made by the infrastructure given as $c_{i,gv}$.  
5. The demand by private entities for capital and inventories given as $c_{i,cp}$.  
6. The exports made outside the set of infrastructures given as $c_{i,exp}$.  
7. The imports procured by the infrastructure to produce its output given as $c_{i,imp}$.  

The I-O model when applied to a set of industries given as $i = \{1,2,\ldots,r\}$, where $r$ is the total number of sectors, is represented as

$$q_i = \sum_{j=1}^{r} a_{ij} q_j + c_{i,hh} + c_{i,gv} + c_{i,cp} + c_{i,exp} - c_{i,imp}$$  \hspace{1cm} (33)$$

$$q_i = \sum_{j=1}^{r} a_{ij} q_j + c_i$$  \hspace{1cm} (34)$$

Where $c_i = c_{i,hh} + c_{i,gv} + c_{i,cp} + c_{i,exp} - c_{i,imp}$. The above equation is represented in matrix form as

$$q = Aq + c \Rightarrow q = [I - A]^{-1}c$$  \hspace{1cm} (35)$$

The above equation measures the balance of supply and demand on the macroeconomic scale assuming a linear demand function. It is assumed that the pre-disruption values for the supply and demand variables are known and denoted by $q^0$, $c^0$. Economic impacts of disruptions are evaluated by considering the changes in the metrics of the above equations. We enumerate the factors that are used in evaluating the metrics immediately after a disruption:

1. After a disruption the production of a sector is affected due to loss of assets resulting in capital loss, loss in labour resulting due to workforce not being able to commute and the loss due to demand. Assuming the pre-disaster production level is given as $q_i^0$, the post-disaster level $q_i$ can be calculated as

$$q_i = \min \left\{ q_i^L, q_i^D, \sum_{j=1}^{r} a_{ij} q_j + c_i \right\}$$  \hspace{1cm} (36)$$

\( q^L_t \) represents the capital of labour, which can be evaluated as

\[
q^L_t = \frac{\text{No. of transport assets affected}}{\text{Total No. of transport assets}} \times q^0_t
\]

\( q^P_t \) represents the production capital, which can be evaluated as

\[
q^P_t = \frac{\text{No. of assets affected}}{\text{Total number of assets}} \times q^0_t
\]

2. The metric \( c_{i,hh} \) depends upon the aggregated demand by all households for the infrastructure output, which is evaluated from data with respect to the pre-disaster household demand levels \( c^0_{i,hh} \).

\[
c_{i,hh} = \frac{\text{No. of households affected}}{\text{Total number of households}} \times c^0_{i,hh}
\]

3. The metrics \( c_{i,gv}, c_{i,cp}, c_{i,exp} \) can be assumed to be unchanged after disruption.

4. \( c_{i,imp} \) depends upon the availability of resources required to transport goods into the economy. If we are able to identify the assets that facilitate transfer of goods into the region then counting the number of such assets which have failed and give us an estimate for the import losses. Assuming the pre-disaster import levels were \( c^0_{i,imp} \) the post-disaster import can be estimated as

\[
c_{i,imp} = \frac{\text{No. of failed import transfer assets}}{\text{Total No. of import transfer assets}} \times c^0_{i,imp}
\]

In the economic I-O loss estimation we are interested in finding the equilibrium state of the disrupted economy, which gives us the new values for the supply and final demands across economic sectors. From the above calculations the disrupted demands and supply can be obtained and if they satisfy the I-O equation then they represent the disrupted equilibrium supply and demand value which can be compared with the pre-disaster levels to obtain economic losses. The problem of bottlenecking\(^{15}\) can occur if the supply falls short of the demand. In particular the inability to satisfy intermediary demands \((Ax)\) results in bottlenecking. This is resolved by employing the following scheme:

1. If \( q_i \geq \sum_{j=1}^n a_{ij} q_j \) then the industry \( i \) is able to provide enough commodity to all other industries and the production of these other industries is not affected.

2. If \( q_i < \sum_{j=1}^n a_{ij} q_j \), then the industry \( i \) is not able to provide enough commodity to all industries and each industry \( j \) sees its production limited by the availability of commodity \( i \). In that case, the production of the industry \( j \) is bounded by

\[
\frac{q_i}{\sum_{j=1}^n a_{ij} q_j} \times q_j
\]

3. The new outputs for industries can be collected as

4. I-O balance is checked and if it is not satisfied then the above 3 steps are repeated again. Finally there is an I-O balance.
6. Implementation of ITRC risk analysis

6.1. Executing the risk calculations

The risk analysis formulations outlined in Section 4 cannot be solved analytically because the exact expressions for the probability density functions are not available as they are data dependent. Also in a real network analysis the size of the problem is large as it is applied to the scale of the entire Great Britain. As such we arrive at answers for Equation (11) by adopting numerical schemes and Monte Carlo simulation techniques.

Based on the different hazard quantifications outlined in Section 5.1.2 there are different numerical approximations to the analytical integrals in the risk calculations. Broadly there are two types of hazard functions that we discussed previously: discrete and continuous. For the discrete distributions the numerical approximations are straightforward, while for the continuous ones sampling techniques are required. Below these calculations for the flooding hazard are shown and the execution steps in the risk analysis are discussed.

6.1.1. Risk analysis using the EA flood hazard estimates

As discussed in Equation (17) the EA flood hazard quantification is qualitative resulting in a discrete distribution. Based on the Figure 2 EA definitions a flooding probability distribution is given as

\[
p(\text{flooding}) = \begin{cases} 
\text{Low} & \text{if } < 1/1000 \\
\text{Medium} & \text{if } 1/1000 - 1/100 \\
\text{High} & \text{if } > 1/100
\end{cases}
\]  \hspace{1cm} (42)

Using this information and in keeping with the notation developed throughout Section 4, the hazard set is given as \( h = \{h_0 = \text{No flooding}, h_1 = \text{Low}, h_2 = \text{Medium}, h_3 = \text{High}\} \) and its distribution can be given as 

\[
f(h_k) = \{1 - \sum_{i=1}^{3} p(h_k), p(h_1), p(h_2), p(h_3)\},
\]

where \( p(h_k) \) denotes the probability of the flooding event. As such the individual asset failure probability in Equation (3) becomes

\[
P(\eta_i) = \sum_{k=1}^{3} L(\eta_i|h_k)p(h_k)
\]  \hspace{1cm} (43)

The overall network failure probability in Equations (5) through (6) is reduced to

\[
P(\bar{f}) = \sum_{j=1}^{n} \sum_{k=1}^{3} \prod_{i=1}^{b} \max\{0, 1 - r_i^{j}\} L(r_i^{j}|h_k), r_i^{j}[1 - L(r_i^{j}|h_k)]\} p(h_k)
\]  \hspace{1cm} (44)

For each of the hazard event the hazard footprint given by \( H(x) = \{H_1(x), H_2(x), H_3(x)\} \) is used to estimate the direct and indirect damage assets and form the set \( \bar{A} \) for each failure mechanism in the set \( \bar{f} \). The direct damage losses \( D_d(r^j) \) (in economic terms) for each mechanism are estimated along with the infrastructure footprint \( S(\bar{A}) \) and the customer effects \( C(\bar{A}, S) \). These estimates are then used in the I-O analysis to find the indirect damage losses and the total damage losses. I-O calculations in Equation (35) give the overall economic damage due to one failure mechanism \( D(r^j) = q \). The overall estimate of network risk is given as
The flowchart in Figure 5 shows the overall risk methodology.

Figure 5: Flowchart showing steps in the risk methodology using the EA flood hazard information.

6.1.2. Risk calculations using the spatial hazard estimates
The risk analysis implementation outlined here applies to the spatially coherent extreme flood depth models and also the rainfall and subsequently G2G models discussed in Section 5.1.2. The analysis is applicable to the other hazards (heat and wind) as well.

Given the climate extreme models being used, an analytical hazard probability distribution for the spatially extreme hazards is not available due to the underlying data assumptions in the hazard models. As such both spatial extreme models and the G2G models provide an ‘event set’ for hazards. In the Equation (14) general continuous hazard distribution the threshold hazard values are important for the hazard quantification. Hence a finite set of threshold levels for the entire topography (or scalar at each asset level) can be used to generate an event set given as

\[ h = \{h_1, h_2, ..., h_t | h_{tr}\} \] (46)
It is assumed that each hazard in the set \( h \) is equally likely to occur. It is also assumed that the hazard distribution given by \( \int_{h_{	ext{tr}}}^{\infty} f(h) \, dh = \mathbb{P}[h \geq h_{\text{tr}}] \) is available to us, so we can get values at the sampling points given by the set in Equation (46). A continuous sampling function \( v(h) \) such that \( \int v(h) \, dh = 1 \) is defined for an importance sampling Monte Carlo solution of the integrals in the risk calculations. The individual asset failure probability in Equation (3) becomes

\[
P(r_i) = \frac{1}{l} \sum_{k=1}^{l} L(r_i| h_k) \frac{f(h_k)}{v(h_k)}
\]

Similar assumptions can be applied to the overall network failure probability in Equations (5) through (6) to obtain

\[
P(\mathbf{r}) = \frac{1}{l} \sum_{j=1}^{n} \sum_{k=1}^{l} \prod_{i=1}^{b} \max \left[ \max[0, 1 - r_i^j] L(r_i^j| h_k), r_i^j [1 - L(r_i^j| h_k)] \right] \frac{f(h_k)}{v(h_k)}
\]

Similar to the process outlined for the EA flood maps analysis for each of the hazard event the hazard footprint given by \( H(\mathbf{x}) = \{H_1(\mathbf{x}), H_2(\mathbf{x}), ..., H_l(\mathbf{x})\} \) is generated used to estimate the direct and indirect damage assets and form the set \( \mathcal{A} \) for each failure mechanism in the set \( \mathbf{r} \). The direct damage losses \( D_d(\mathbf{r}^j) \) (in economic terms) for each mechanism are estimated along with the infrastructure footprint \( \mathcal{S}(\mathcal{A}) \) and the customer effects \( \mathcal{C}(\mathcal{A}, \mathcal{S}) \). These estimates are then used in the I-O analysis to find the indirect damage losses and the total damage losses. I-O calculations in Equation (35) give the overall economic damage due to one failure mechanism \( D(\mathbf{r}^j) = q \). The overall estimate of network risk is given as

\[
R(\mathbf{r}) = \frac{1}{l} \sum_{j=1}^{n} D(\mathbf{r}^j) \sum_{k=1}^{l} \prod_{i=1}^{b} \max \left[ \max[0, 1 - r_i^j] L(r_i^j| h_k), r_i^j [1 - L(r_i^j| h_k)] \right] \frac{f(h_k)}{v(h_k)}
\]

It is assumed in the above calculations that some estimate of the hazard distribution is available to us. But in general the hazard is reported in terms of the return period or the exceedance probability. In such a case the fragility calculation is given by

\[
P(r_i) = \sum_{k=2}^{l} L(r_i| h_k)(\mathbb{P}[h_k \geq h_{\text{tr}}] - \mathbb{P}[h_{k-1} \geq h_{\text{tr}}])
\]

\[\text{In a particular case of the above calculation, if } \{h_1 > \cdots > h_i\} \text{ then the Monte Carlo sum approximation for the integral } \int_{h_1}^{h_i} f(h) \, dh \text{ becomes } \sum_{k=1}^{h_i} f(h_k) \left( h_i - h_k \right) = \frac{1}{l} \sum_{k=1}^{h_i} f(h_k) \frac{1}{h_i - h_k}, \text{ which means } v(x) \text{ could be a uniform distribution in the range }[h_1, h_i]. \text{ In general a distribution which is close to } f(h) \text{ is chosen.}\]

\[\text{The hazard footprint for flooding are generally generated use the EA flood maps and using interpolation methods to obtain spatial extent maps corresponding to the locations where flows (and depths) are estimated.}\]
Here $f(h)$ is replaced by the approximation $p(h_k) = \mathbb{P}[h_k \geq h_{\text{TR}}] - \mathbb{P}[h_{k-1} \geq h_{\text{TR}}]$ which takes the difference between different exceedance probabilities$^{18}$. Similar substitutions are made in the network failure probability and risk calculations.

A flowchart shown in Figure 6 summarises the steps in the risk methodology.

Figure 6: Flowchart showing steps in the risk methodology using the probabilistic and spatial hazard information.

### 6.2. Workflows and Phases in ITRC

The ITRC risk analysis framework is summarised in the following three flowcharts, which highlight the workflow, tasks and the responsibilities of the different teams leading the work. As mentioned previously ITRC WS2 risk analysis will be implemented in three steps or Phases.

#### 6.2.1. Phase 1

Phase 1, shown in Figure 7, is an infrastructure vulnerability assessment framework that uses flood extent maps and binary connectivity among infrastructures to assess the damage and losses for assets affected by flooding. Outcomes of this analysis provide a count of the failed

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assets within and outside the flood plains, infrastructure footprint measures, the customers affected by the loss of services and the resulting economic losses. While most of the steps in the flowchart are self-explanatory, we provide their link with the notations in risk methodology below.

1. **Hazard estimation (Component A):** Based on EA flood plain maps and Cranfield NPD flood extent maps we obtain estimates for \( H(\mathbf{x}) \). The EA flood maps provide flood extents for 3 return periods: (i) a low probability event with 1/1000 event probability, (ii) a moderate probability event with 1/500 event probability, and (iii) a high probability with 1/100 event probability. These maps are provided for Great Britain, and initially used for analysis over England. The Cranfield flood maps provide alluvial floodplain outlines based on geological soil conditions of areas which were once lakes, rivers or sea. Similar to the EA maps such maps are being used at the England level. The maps are national, but aggregates of vulnerability can be generated at regional or catchment scales. They do not represent spatially coherent events so care needs to be taken in presenting and interpreting the results.

2. Based on data supplied by the ITRC WS4 database we collect the data required for generating a spatial set \( A \) of assets. Asset data exist for the following infrastructures:
   a. Electricity Sub Stations
   b. Energy Production sites
   c. Refineries
   d. Telco Masts
   e. Water Pumping Stations
   f. Waste Water Treatment Plants
   g. Roads
   h. Railway lines
   i. Train Stations
   j. Airports
   k. Ports

Such data is provided by the Ordnance Survey for each asset type across England. The data is collected by the ITRC team in Newcastle University lead by David Alderson and processed by Scott Thacker at Oxford University.

3. Flood plain and asset intersection produces the failure mechanism set \( \bar{f} \), and the set of directly failed assets \( A_{\text{fail}}^{\text{dir}} \). The underlying assumption in the initial analysis is that the assets within the flood plain have a probability of failure equal to one, which means the fragility of the asset is equal to one. Hence the set \( A_{\text{fail}}^{\text{dir}} \) contains all those assets that with certain failure, which has been termed as a first-order failure mechanism. While this is a similar to a worst-case analysis of failure, it can be further refined, by sensitivity testing that reduces the probability of asset failure. Such analysis is performed by Scott Thacker at Oxford University.
4. Interdependency mechanisms are generated to develop a crude network representation to produce the set \( \{N, E, M\} \). The interdependency mechanisms considered are physical and geographic. Physical interdependency exists due to the reliance on material flow from one infrastructure to another. Geographic interdependency exists due to local environmental event affects across multiple infrastructures within locational proximity of the map. Based on the set of assets available interdependency is derived from the notion that two different infrastructure assets exchange goods and services between themselves, and geographically close to each other. For assets satisfying the above criteria the ‘edge’ in the set \( E \) is assigned a value of 1 showing interdependency exists between assets. Table 4 is derived for the types of assets being used in the Phase 1 analysis. Using this table the interdependency rules adopted in finding damage propagation from an asset type \( A \) to an asset type \( B \) is derived as follows:

a. Asset \( A \) fails and if \( B \) is physically dependent upon \( A \) and within a specified geographic radius of \( A \), then \( B \) also fails.

b. Asset \( A \) fails and if \( B \) is physically dependent upon \( A \) but outside a specified geographic radius of \( A \), then \( B \) does not fail.

c. Asset \( B \) fails and if \( B \) is physically dependent upon \( A \), then \( A \) does not fail.

It is to be noted here that the interdependency is considered across different types of infrastructure assets and is neglected for assets of the same type. This work is also being conducted by Scott Thacker.

Table 4: List of infrastructure assets considered in Phase 1 with interdependency structure

<table>
<thead>
<tr>
<th>Infrastructure Asset Has a dependency on…</th>
<th>Substation</th>
<th>Energy production</th>
<th>Refinery</th>
<th>Telco Mast</th>
<th>Water Pumping Station</th>
<th>Waste Water Treatment Works</th>
<th>Roads</th>
<th>Railway Lines</th>
<th>Train Stations</th>
<th>Airports</th>
<th>Ports</th>
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<tr>
<td>Substation</td>
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<td>Refinery</td>
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<td>Telco Mast</td>
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<td>Water Pumping Station</td>
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<td>Train Stations</td>
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<td>Airports</td>
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<td>Ports</td>
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5. **Asset damage assessment (Component B):** From the set of directly failed assets (Step 3) and the interdependency mechanisms (Step 4) are able to generate the set of indirectly failed assets $A_{\text{indir}}^{\text{fail}}$ and the overall asset failure set $\tilde{A}$. In finding the indirectly failed assets the geographic proximity of assets is also used as criteria along with the physical interdependence to identify further failed assets. The failure mechanism, called a second-order failure mechanism, is executed in the following steps:
   a. Choose an asset that lies in the flood-plain and is designated to have failed already.
   b. Identify all other infrastructure assets within a specified distance from the chosen asset that are physically interdependent on the failed asset.
   c. Assign the identified assets to be ‘failed’ by virtue of not being able to function due to loss of resources from the directly failed asset.

As mentioned in point 4, the above analysis includes just a direct version of interdependence. No networks effects are considered. This analysis is also conducted by Scott Thacker.

6. Infrastructure failure footprint calculation is made to generate the set $\tilde{S}$ for identifying the area influence of damaged assets. This uses demographic data provided at the district or ward levels which gives population boundaries for analysis. The Office of National Statistics (ONS) Output Areas (OA) represents the lowest level of spatially explicit population data in England at the ward level given by postal codes. Collectively the individual areas cover the whole land area of England, forming a set $S$ of polygon areas containing populations. Initially, due to lack of explicit customer demand data associated with each asset, the population figures are utilised to find the maximum number of customers within an area surrounding an asset. In order to construct failure footprints and estimate damages to customers it is first necessary to assign demand areas (with associated customer estimates) to individual assets. The failure footprint set $\tilde{S}$ is generated by collecting all the area polygons surrounding the failed assets. For energy and water assets we generate Thiessen polygons surrounding each distribution asset. Thiessen polygons define individual areas around assets such that each point of the boundary of the areas polygon is closest to the given asset. The polygons serve as suitable representation of the customers served by assets such as energy substations and water pumping stations. Hence for these types of assets the steps in finding $\tilde{S}$ are as follows:
   a. Chose a failed asset from the set $\tilde{A}$.
   b. Find the thiessen polygon area around the failed asset.
   c. Do the same for more assets, while checking that two assets do not have the same polygons of influences.
While this method is an approximate estimate for obtaining failure footprint, it is nonetheless a useful initial exercise. It can be further refined by adding weights to the areas polygon based on the output capacities of the assets (for example substation operating at 33kV has a bigger footprint than substations at 11kV). Also information of the polygon areas for higher networks at the transmission level can be utilised to create larger influence areas for infrastructures and these can be divided into smaller thieissen polygons. For transportation assets such as ports and airports the footprints are measured in terms of the number of people moving through these hubs, which translates to loss of flow when failure occurs. Transport assets such as roadways and rail lines require assessment of the number of people travelling in given time and assessing the loss of travel distance and travel time across routes when failures occur. More accurate failure footprints can be obtained if there is better data (obtained through stakeholder engagement) that gives the area serviced by each infrastructure asset. Such analysis of also conducted by Scott Thacker at Oxford University.

7. **Infrastructure damage assessment (Component C):** Using the sets $\tilde{A}$ (Step 5) and $\tilde{S}$ (Step 6) we can generate the customer catchment estimates $\tilde{C}(\tilde{A}, \tilde{S})$ to obtain the number of customers affected by infrastructure failure. The set $\tilde{A}$ also gives a count of the number of assets of each type that fail. Hence this information is used to estimate the direct impact effects for asset damages $D_d(\tilde{r}_i^j)$ and network losses $D_d(\tilde{r})$. For economic analysis purposes, the following effects constitute direct damage impacts due to infrastructure asset failures:
   a. The failure of the assets directly due to the flooding or due to interdependence. The direct impact of such failure is that the assets cannot produce outputs.
   b. The customers that represent households in the set $\tilde{C}$ who are affected due to the loss of resources from the failed assets.
   c. The customers that represent businesses in the set $\tilde{C}$ that are affected due to the loss of resources from the failed assets.
   d. The customers that represent government interests in the set $\tilde{C}$ that are affected due to the loss of resources from the failed assets.

This analysis is conducted by Scott Thacker and Raghav Pant at Oxford University.

8. This analysis is also conducted by Scott Thacker and Raghav Pant at Oxford University. The asset and network loss estimates in Step 7 are used as supply and demand loss inputs respectively for an economic input-output analysis calculation. These metrics of supply and demand are estimates at the national England level in the Phase 1 analysis. For assets are distributed throughout the geography of England the economic impacts of failures are assessed by aggregating each asset type over the entire country level. Hence, rather than being concerned with the failure impact of an individual asset, we are interested in aggregated national economic impact of all assets that fail. The direct damage effects which we listed in Step 7 become inputs for the economic analysis as follows:
   a. The loss of outputs of assets is a loss of supply. If for each asset the output is known then we can estimate the output loss it incurs. For example the
economic value of machinery, capital and output of an electricity substation or water pumping station, etc. gives the loss of supply for the asset. By adding over all similar asset types we can get the supply loss at the aggregated national level. In the absence of sufficient data, another method of estimating supply loss for an infrastructure type is by calculating the fraction of its failed assets out of its total assets. Calculations are done for the assets in Table 1.

b. The losses for household customers are demand losses, which are also aggregated over the national England scale. Apart from the transportation infrastructures all other infrastructure types produce resources demanded by households. Hence for electricity substations the demands are measured in terms of the electricity bills of the customers consuming electricity. Similarly water and waste bills, gas bills, phone bills for households give estimates for the household demands from such infrastructure types. Aggregated over the national England scale estimates can be obtained for demand losses for households. In the absence of sufficient data, again such losses can be estimated by considering the fraction of households affected out of the total households in the country.

c. The losses for business customers are also demand losses and can be obtained in a similar method to the household demand loss calculations.

d. The losses for government customers are also demand losses and can be obtained in a similar method to the household demand loss calculations.

Table 5: Supply side and demand side metrics generated for each infrastructure asset type that fails

<table>
<thead>
<tr>
<th>Infrastructure Asset</th>
<th>Asset damage (supply side)</th>
<th>Household customers (demand side)</th>
<th>Business customers (demand side)</th>
<th>Government customers (demand side)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substation</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Energy production</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Refinery</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Telco Mast</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Water Pumping Station</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Waste Water Treatment Works</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Roads</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Railway Lines</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Train Stations</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Airports</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Ports</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

9. **Economic damage assessment (Component D):** This analysis is conducted by the ITRC team at Cambridge University lead by Scott Kelly. The supply and demand side loss inputs generated in Step 8 are fed into an economic input-output model. The economic model being used consists of 87 infrastructures or economic sectors as they
are known in the input-output model. The infrastructures in Table 1 are present among of 87 sectors or can be related closely to some sectors in the list. Using the economic input-output analysis to find the disrupted equilibrium state we can generate the indirect losses $D_{in}(F)$ and total loss $D(F)$ due to the network. The outputs of such an analysis are the direct and indirect output losses for individual economic sectors and the total economic losses in across all sectors. Other metrics like amplification ratios, which show the ratios of direct/indirect losses for sectors are also generated in the analysis.

Figure 7: Flowchart showing steps in the ITRC WS2 Phase 1 (First-Pass) infrastructure vulnerability analysis.

6.2.2. Phase 2
Phase 2, shown in Figure 8, is an infrastructure risk analysis framework that uses probabilistic hazard models and extent maps and topological connectivity among infrastructures to assess the damage and losses for assets affected by climate hazards. Outcomes of this analysis provide infrastructure fragility values, a count of the failed assets within and outside the flood plains, infrastructure footprint measures, the customers affected by the loss of services, the resulting economic losses, and overall risk calculations. Future climate and infrastructure scenarios will further be used to get estimates of long-term risks of failures of infrastructures. The different tasks in these flowcharts are related to the notations in risk methodology as follows:
1. **Hazard estimation (Component A):** Based on different types of hazard quantification (Hazard extent maps/Spatial distributions of hazards/Spatial-temporal distributions of hazards) we obtain estimates for \( f(h) \) (or \( p(h_k) \)) and \( H(h) \). The different hazard quantification schemes employed in the analysis are:
   a. The EA flood maps for 3 return periods: (i) a low probability event with 1/1000 event probability, (ii) a moderate probability event with 1/500 event probability, and (iii) a high probability with 1/100 event probability. These maps are provided for Great Britain.
   b. The Cranfield flood maps provide alluvial floodplain outlines based on geological soil conditions of areas which were once lakes, rivers or sea. Such maps are being used at the Great Britain level.
   c. The JBA spatially coherent models that provide measures for extreme river or sea flood flows (and depths) from which the joint probability of extreme river flows or sea levels at multiple locations reported in terms of the return periods. Hazard footprint measures can also be generated here by interpolation across gauge locations to obtain the comparable flood hazard footprint estimates. Five event set maps, similar to the EA flood maps, have been generated for Great Britain.
   d. The Newcastle University team of Chris Kilsby generate rainfall models using daily rainfall data to generate continuous time-series spatially coherent rainfall estimates, which generate a climate ensemble of a few hundred events. Each rainfall event is generated on a 25km gridded area of the map, which can be disaggregated to 5km grid levels using weighting methods. Using the precipitation input data the grid-to-grid (G2G) rainfall flow routing and runoff-production model of CEH generates estimates of river flows and fluvial discharges into the sea. The analysis generates flow estimates (which might be further processed to get flood depths) on 1km (or 5 km or 25 km) grids on the entire map over river catchment areas. The G2G model can be used to provide flow estimates over a large area, which would lead to hazard footprint estimates. Present and future rainfall climate scenario input makes such analysis useful for assessing future risks to infrastructures.
   e. The Newcastle climate modelling team of the ITRC is also engaged in modelling other climate hazards such as wind and heat for the Great Britain.

The maps generated in all the above models are national, but aggregates of risk can be generated at regional or catchment scales.

2. **Network estimation (Component B):** Based on data supplied by the ITRC WS4 database and WS1 future network scenarios we collect the data required for generating the asset set \( A \) in the present and the future. Further, the interdependency mechanisms are used to generate a topological network representation to produce the set \( \{N, E, M\} \). The network is updated based on future scenarios. This work is conducted by David Alderson of the ITRC team in Newcastle University. Tables 3 to Table 7 provide a summary of the asset elements that make up each infrastructure network. Data for these is provided through central agencies like the Ordnance Survey.
and individual infrastructure stakeholders like National grid, at the lowest possible level of aggregation for the entire Great Britain.

Table 6: List of assets and scale for the energy infrastructure

<table>
<thead>
<tr>
<th>Energy networks</th>
<th>Network components</th>
<th>Network characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electricity</td>
<td><strong>Generation sites</strong>: Coal, nuclear, wind, hydro, oil, CCS, Pumped storage, CCGT, Dual fuel, CHP</td>
<td>Network nodes spread nationally</td>
</tr>
<tr>
<td></td>
<td><strong>Transmission</strong>: High voltage wires</td>
<td>Network edges spread nationally</td>
</tr>
<tr>
<td></td>
<td><strong>Distribution</strong>: Substations, low voltages wires</td>
<td>Network nodes and edges spread at regional levels</td>
</tr>
<tr>
<td>Gas</td>
<td><strong>Storage sites</strong>: Terminals, Compressors, storage, LNG</td>
<td>Network Nodes spread nationally</td>
</tr>
<tr>
<td></td>
<td><strong>Transmission</strong>: High pressure pipes</td>
<td>Edges spread nationally</td>
</tr>
<tr>
<td></td>
<td><strong>Distribution</strong>: Refinery sites and low pressure pipes</td>
<td>Network nodes and edges spread regionally</td>
</tr>
</tbody>
</table>

Table 7: List of assets and scale for the transport infrastructure

<table>
<thead>
<tr>
<th>Transport networks</th>
<th>Network components</th>
<th>Network characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Road</td>
<td><strong>Crossroads and intersections</strong></td>
<td>Network nodes spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td><strong>Road</strong>: Motorways, minor roads, A roads, B roads</td>
<td>Network edges spread nationally and regionally</td>
</tr>
<tr>
<td>Rail</td>
<td><strong>Train stations and junctions</strong></td>
<td>Network Nodes spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td><strong>Rail lines, bridges</strong></td>
<td>Edges spread nationally and regionally</td>
</tr>
<tr>
<td>Water</td>
<td><strong>Terminals</strong>: Ferry transport points, Freight transfer ports</td>
<td>Network Nodes spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td><strong>Navigable waterways</strong>: Lakes, rivers</td>
<td>Edges spread nationally and regionally</td>
</tr>
<tr>
<td>Air</td>
<td>Airports</td>
<td>Network Nodes spread nationally and regionally</td>
</tr>
</tbody>
</table>
Table 8: List of assets and scale for the water infrastructure

<table>
<thead>
<tr>
<th>Water networks</th>
<th>Type</th>
<th>Network components</th>
<th>Network characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>Storage: Lakes, reservoirs, ground water</td>
<td>Network nodes spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Treatment: Pumping stations</td>
<td>Network nodes spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distribution: Pipes</td>
<td>Network edges spread nationally and regionally</td>
</tr>
</tbody>
</table>

Table 9: List of assets and scale for the waste infrastructure

<table>
<thead>
<tr>
<th>Waste networks</th>
<th>Type</th>
<th>Network components</th>
<th>Network characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wastewater</td>
<td>Treatment: Pumping stations, sewage treatment plants</td>
<td>Network nodes spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distribution: Pipes and channels for effluent discharges</td>
<td>Network edges spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td>Solid waste</td>
<td>Terminals: collection points, treatment points, recycling centres, disposal points</td>
<td>Network Nodes spread nationally and regionally</td>
</tr>
</tbody>
</table>

Table 10: List of assets and scale for the ICT infrastructure

<table>
<thead>
<tr>
<th>ICT networks</th>
<th>Type</th>
<th>Network components</th>
<th>Network characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Telecommunications</td>
<td>Towers: Masts for receiving signals</td>
<td>Network nodes spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distribution: Fibre optic cables, telephone cables</td>
<td>Network edges spread nationally and regionally</td>
</tr>
<tr>
<td></td>
<td>Data centres</td>
<td>Terminals: Point for storing and processing information</td>
<td>Network Nodes spread nationally and regionally</td>
</tr>
</tbody>
</table>

3. Hazard and asset intersection produces the failure mechanism set \( \bar{F} \), the set of directly failed assets \( A_{f_{\text{fail}}}^{\text{dir}} \), the set of indirectly failed assets \( A_{f_{\text{fail}}}^{\text{indir}} \), and the overall asset failure set \( \bar{A} \). The failure mechanism hinges of generating the direct failure set \( A_{f_{\text{fail}}}^{\text{dir}} \) because the rest of the mechanism comes through the connectivity. As such an event tree type of analysis is used to generate possible sets of assets that fail directly due to the hazard. The severity of the hazard and the assets connectivity are used as factors.
in generating possible mechanisms. Such analysis is performed by Raghav Pant at Oxford University.

4. **Network reliability analysis (Component C):** Using the asset and network failure sets in Step 3 we can generate asset fragility $L(r_i|\{h\})$ and the network failure probability $P(\mathbf{F})$. Given that there is only knowledge of the asset connectivity at this level the assets within the flood plain are assigned probability of failure depending upon the severity of the hazard and their connectivity, which is used in calculating the fragility of the asset. Network connectivity measures such as node centrality can be used to estimate asset fragility. Such analysis is performed by Raghav Pant at Oxford University.

5. Infrastructure failure footprint calculation is made to generate the set $\mathcal{S}$ for identifying the area influence of damaged assets. In order to construct failure footprints and estimate damages to customers it is first necessary to assign demand areas (with associated customer estimates) to individual assets. The failure footprint set $\mathcal{S}$ is generated by collecting all the area polygons surrounding the failed assets. As outlined in the Step 6 of the Phase 1 methodology we can also use the population boundaries data provided at the ward levels by the Office of National Statistics (ONS) Output Areas (OA) representing the lowest level of spatially explicit population data in Great Britain. The population figures are utilised to find the maximum number of customers within an area surrounding an asset. More accurate failure footprints can be obtained if there is better data (obtained through stakeholder engagement) that gives the area serviced by each infrastructure asset. Such analysis of also conducted by Raghav Pant at Oxford University.

6. **Infrastructure damage assessment (Component D):** Using the sets $\mathcal{A}$ (Step 5) and $\mathcal{S}$ (Step 6) we can generate the customer catchment estimates $\mathcal{C}(\mathcal{A},\mathcal{S})$ to obtain the direct impact effects for asset damages $D_d(r_i^l)$ and network losses $D_d(\mathbf{F})$. For economic analysis purposes, the following effects constitute direct damage impacts due to infrastructure asset failures:
   a. The failure of the assets directly due to the flooding or due to interdependence. The direct impact of such failure is that the assets cannot produce outputs.
   b. The customers that represent households in the set $\mathcal{C}$ who are affected due to the loss of resources from the failed assets.
   c. The customers that represent businesses in the set $\mathcal{C}$ that are affected due to the loss of resources from the failed assets.
   d. The customers that represent government interests in the set $\mathcal{C}$ that are affected due to the loss of resources from the failed assets.
   e. The loss of labour due to failure of transport infrastructure assets.
   f. The loss of exports and imports due to failure of infrastructure assets.

This analysis is conducted by Raghav Pant at Oxford University.

7. This analysis is also conducted by Raghav Pant at Oxford University. The asset and network loss estimates in Step 6 are used as supply and demand loss inputs respectively for an economic input-output analysis calculation. These metrics of
supply and demand are estimates at the Great Britain level. For assets are distributed throughout Great Britain the economic impacts of failures are assessed by aggregating each asset type over the entire country level. Hence, rather than being concerned with the failure impact of an individual asset, we are interested in aggregated national economic impact of all assets that fail. The direct damage effects which we listed in Step 6 become inputs for the economic analysis as follows:

a. The loss of outputs of assets is a loss of supply. If for each asset the output is known then we can estimate the output loss it incurs. For example the economic value of machinery, capital and output of an electricity substation or water pumping station, etc. gives the loss of supply for the asset. By adding over all similar asset types we can get the supply loss at the aggregated national level. In the absence of sufficient data, another method of estimating supply loss for an infrastructure type is by calculating the fraction of its failed assets out of its total assets.

b. The losses for household customers are demand losses, which are also aggregated over the Great Britain scale. Apart from the transportation infrastructures all other infrastructure types produce resources demanded by households. Hence for electricity substations the demands are measured in terms of the electricity bills of the customers consuming electricity. Similarly water and waste bills, gas bills, phone bills for households give estimates for the household demands from such infrastructure types. Aggregated over the national Great Britain scale estimates can be obtained for demand losses for households. In the absence of sufficient data, again such losses can be estimated by considering the fraction of households affected out of the total households in the country.

c. The losses for business customers are also demand losses and can be obtained in a similar method to the household demand loss calculations.

d. The losses for government customers are also demand losses and can be obtained in a similar method to the household demand loss calculations.

e. The loss of labour due to failure of transport infrastructure assets is a supply side loss because it results in an inability of the infrastructure to generate its output. An estimate of the number of people working at a failed facility and the number using a road, rail, airport or port leading to that facility can provide an estimate of the loss of labour for that asset. By aggregating over the total such transport failures and the commuter losses we can estimate the overall national level labour losses due to the infrastructures. Again in the absence of sufficient data, an estimate of the fraction of damaged roads, rail lines, airports or ports gives an estimate of overall labour losses.

f. The loss of exports and imports due to failure of transport infrastructure assets are demand losses. The failure of the assets that link to regions outside England provides an estimate for such losses. If for each infrastructure data for exports and imports are available then the losses can be estimated, otherwise a
8. Economic damage assessment (Component D): This analysis is conducted by the ITRC team at Cambridge University lead by Scott Kelly. The supply and demand side loss inputs generated in Step 7 are fed into an economic input-output model. The economic model being used consists of 87 infrastructures or economic sectors as they are known in the input-output model. The infrastructures in Tables 3-7 are present among of 87 sectors or can be related closely to some sectors in the list. Using the economic input-output analysis to find the disrupted equilibrium state we can generate the indirect losses $D_{in}(\bar{F})$ and total loss $D(\bar{F})$ due to the network. The outputs of such an analysis are the direct and indirect output losses for individual economic sectors and the total economic losses in across all sectors. Other metrics like amplification ratios, which show the ratios of direct/indirect losses for sectors are also generated in the analysis.

9. Risk calculation (Component F): This analysis is conducted by Raghav Pant at Oxford University. By collecting the damage estimated in the Step 8 and the reliability estimates in the Step 4 above the estimates for asset risk and network risk $R(\bar{F})$ are generated.

---

**Figure 8:** Flowchart showing steps in the ITRC WS2 Phase 2 infrastructure risk analysis.
6.2.3. Phase 3

Phase 3, shown in Figure 9, is an infrastructure risk analysis framework that uses probabilistic hazard models and extent maps with topological networks having flow/flux attributes to assess the flow redistributions, damage and losses for assets affected by climate hazards. Outcomes of this analysis provide infrastructure fragility values, a count of the failed assets within and outside the flood plains, infrastructure footprint measures, the customers affected by the loss of services, the resulting economic losses, and overall risk calculations. All these calculations are spatial and temporal with flow redistribution characteristics that also provide metrics for network resilience. Estimates of future climate scenarios and infrastructure capacity and demand modelling\(^{19}\) provide long-term dynamic estimates of infrastructure risk. The steps and task responsibilities in the execution of the Phase 3 methodology are similar to Phase 2 with difference arising in the nature of the network characterisation and analysis and the economic analysis. The different tasks in these flowcharts are related to the notations in risk methodology as follows:

1. **Hazard estimation (Component A):** Based on different types of hazard quantification (Hazard extent maps/Spatial distributions of hazards/Spatial-temporal distributions of hazards) we obtain estimates for \( f(h) \) (or \( p(h_k) \)) and \( H(h) \). This was explained in Step 1 of the Phase 2 methodology.

2. **Network estimation (Component B):** Based on data supplied by the ITRC WS4 database and WS1 CDAM analysis we collect the data required for generating the asset set \( A \) in the present and the future. The data includes asset properties like capacity, flow and flux that add to the topology. This information is combined with the interdependency mechanisms to generate a topological capacity and flow based network representation to produce the set \( \{N, E, M\} \). The network is updated based on future scenarios. As explained in the Phase 2 Step 2 the ITRC team in Newcastle University will construct the topological infrastructure networks and further incorporate asset attributes such as capacity, flow and fluxes.

3. Hazard and asset intersection produces the failure mechanism set \( \bar{A} \), the set of directly failed assets \( A_{\text{dir}}^{\text{fail}} \), the set of indirectly failed assets \( A_{\text{indir}}^{\text{fail}} \), and the overall asset failure set \( \bar{A} \). Due to the availability of asset properties we can have a better estimate of the failures because the ability of each asset to sustain loading is known, which helps identify failed assets. Such analysis is conducted by Raghav Pant at Oxford University.

4. **Network reliability analysis (Component C):** Using the asset and network failure sets in Step 3 we can generate asset fragility \( L(r_i|h) \) and the network failure probability \( P(\bar{f}) \). Given our knowledge of the asset capacity and the external loading, we can estimate fragility in its true sense now. This analysis also conducted by Raghav Pant at Oxford University.

5. Infrastructure failure footprint calculation is made to generate the set \( \tilde{S} \) for identifying the area influence of damaged assets. The similar approaches outlined in Phase 2 calculations are employed here, given the availability of appropriate data. This analysis also conducted by Raghav Pant at Oxford University.

6. **Infrastructure damage assessment (Component D):** Using the sets \( \tilde{A} \) (Step 5) and \( \tilde{S} \) (Step 6) we can generate the customer catchment estimates \( \tilde{C}(\tilde{A}, \tilde{S}) \) to obtain the direct impact effects for asset damages \( D_d(\gamma^j) \) and network losses \( D_a(\bar{r}) \). The analysis is similar to the Phase 2 economic analysis and is conducted by Raghav Pant at Oxford University.

7. The asset and network loss estimates in Step 6 are used as supply and demand loss inputs respectively for an economic input-output analysis calculation. Such data consists of the present and future cost-adaption scenarios which would update the input-output technical coefficient matrix \( A \). The analysis is similar to the Phase 2 economic analysis and is conducted by Raghav Pant at Oxford University.

8. **Economic damage assessment (Component D):** Using the economic input-output analysis to find the disrupted equilibrium state we can generate the indirect losses \( D_{in}(\bar{r}) \) and total loss \( D(\bar{F}) \) due to the network. The analysis is similar to the Phase 2 economic analysis and is conducted by Scott Kelly at Cambridge University.

9. **Risk calculation (Component F):** This analysis is conducted by Raghav Pant at Oxford University. By collecting the damage estimated in the Step 8 and the reliability estimates in the Step 4 above the estimates for asset risk and network risk \( R(\bar{F}) \) are generated.

10. The Steps 3 to 9 are repeated as the network flows redistribute over time and space to provide another iteration of the risk calculation.
6.2.4. Summary of tasks and responsibilities

Below is a summary of the tasks and responsibilities of the different teams involved in the ITRC WS2 analysis Phases. The tables below show the work of each team in the different phases with the inputs they receive, the task they perform and the output they generate.
<table>
<thead>
<tr>
<th>Team/Agency</th>
<th>Phase</th>
<th>Input</th>
<th>Task/Tools</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment Agency (EA)</td>
<td>1,2,3</td>
<td>- Land topography</td>
<td>- Generate flood catchment areas</td>
<td>- Extent of extreme flooding for England and Wales scalable at regional levels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- River flow data</td>
<td>- Generate flood maps</td>
<td>- Probability of flood event</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Sea level + wave data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JBA (Rob Lamb/Paige Garside)</td>
<td>2,3</td>
<td>- Gauge river flow data with maximum mean daily flow</td>
<td>- Use statistical conditional exceedance model</td>
<td>- Synthetic spatially coherent (and temporally dependent) flood flows and depths from joint distribution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Sea level data</td>
<td>- Find joint probability of extreme river flow and sea levels at multiple locations</td>
<td>- Extreme flood extent maps for Great Britain and regional scales with different return periods</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Land topography</td>
<td>- Interpolate flood extents from EA</td>
<td></td>
</tr>
<tr>
<td>Cranfield University NPD analysis (Stephen Hallette, Timothy Farewell, Jacqueline Hannam, RA)</td>
<td>1,2,3</td>
<td>- Land topography</td>
<td>- Infer tell-tale signs of past flood events</td>
<td>- Flood extent maps for Great Britain scalable at regional levels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Riverine, lacustrine and marine/coastal flood provinces defined by geological alluvial soil conditions based on sediment deposits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Newcastle University (Chris Kilsby, Francesco Serinaldi)</td>
<td>2,3</td>
<td>- UK Met Office 5 km continuous daily rainfall data from 1958-2008</td>
<td>- Use at-site univariate distribution model to generate discrete-continuous rainfall distribution</td>
<td>- Continuous daily rainfall series on 5 km spatially correlated grids across entire Great Britain</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Annual daily rainfall maximum time series for approximately 200 stations</td>
<td>- Use spatial-temporal random filed model to generate spatially correlated and short-term auto-correlated rainfall estimates</td>
<td>- Monte Carlo event sets for extreme rainfall threshold exceedance on 5 km spatially correlated grids across entire Great Britain</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Geophysical climate conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CEH (Vicky Bell, Alison Kay)</td>
<td>2,3</td>
<td>- UK Met Office 5 km continuous daily rainfall data</td>
<td>- Use Grid-to-Grid (G2G) model for rainfall flow routing and runoff production to generate river flow estimates and fluvial discharges in the sea</td>
<td>- Flow predictions on the 1km G2G grid and corresponding return period for each member of the spatial event set for entire Great Britain scalable at regional levels</td>
</tr>
<tr>
<td>Team/Agency</td>
<td>Phase</td>
<td>Input</td>
<td>Task/Tools</td>
<td>Output</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-------</td>
<td>----------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| Newcastle University        | 1,2,3 | • Ordnance Survey Infrastructure Asset layers on Energy, Transport, Water, Waste and ICT  
• National Grid electricity and gas national transmission system layers  
• DECC Operational power stations data  
• Electricity North West regional electricity distribution data  
• Northern Electric Distribution Limited North East distribution data  
• Urban Waste water treatment data  
• Other sources (in collection process) | • Compile a National Infrastructure Asset Dataset (Phases 1,2,3)  
• Perform series of spatial processing steps create topologically valid representations of the national infrastructure networks (Phases 2,3)  
• Build interdependencies that exist between components of the different networks (Phases 2,3)  
• Incorporate attributes of capacity, flow and fluxes to network components | • Spatially distributed assets across England (Phase 1)  
• Topologically and spatially valid representation of infrastructure networks for Great Britain (Phases 2,3)  
• Topological capacity and flow networks for Great Britain (Phases 3) |
## Oxford University (Jim Hall, Raghav Pant, Scott Thacker)

<table>
<thead>
<tr>
<th>Phase</th>
<th>Input</th>
<th>Task/Tools</th>
<th>Output</th>
</tr>
</thead>
</table>
| 1     | - EA flood extent maps for England  
       - NPD Flood extent maps for England  
       - Government Census population data for district/ward level with output areas  
       - Demand estimates in British Pounds (£) for infrastructure assets resources | - Intersect hazards and infrastructure assets  
       - Develop interdependency mechanisms in failure  
       - Find directly damage assets which lie within floodplain  
       - Find indirectly damaged assets from interdependency relationships  
       - Find spatial extent of damage impacts due to asset failures  
       - Find number of customers affected due to asset damage | - Analysis for England  
       - Number of assets failed due to direct flood damage, indirect interdependency damage, and both effects  
       - Spatial extent of asset damage as polygon area extents around assets  
       - Number of customers affected due to asset failure  
       - Aggregated England level estimates for (of sectors in input-output model consideration)  
         - Total damage in £ for asset damage  
         - Loss of household customer demand in £  
         - Loss of business customer demand in £  
         - Loss of government demand in £ |
| 2     | - EA flood maps and event probabilities  
       - NPD flood extent maps  
       - JBA spatially coherent flood depths and probabilistic flood maps  
       - CEH G2G flow estimates and return periods  
       - Wind, heat hazard model estimates  
       - Topologically and spatially valid representation of infrastructure networks for Great Britain  
       - Government Census population | - Intersect hazards and infrastructure assets  
       - Determine asset fragility  
       - Develop mechanisms in failure  
       - Find directly damage assets and indirectly damaged assets from interdependency relationships  
       - Find spatial extent of damage impacts due to asset failures  
       - Find number of customers affected due to asset damage | - Analysis for Great Britain  
       - Number of assets failed due to direct hazard impact, indirect interdependency impact  
       - Asset and network probability of failures  
       - Spatial extent of asset damage as polygon area extents around assets  
       - Number of customers affected due to asset failure  
       - Number of work hours lost due to transport damages  
       - Number of export-import linkages affected for infrastructures |
<table>
<thead>
<tr>
<th>Data and Analysis</th>
<th>Aggregated Great Britain level estimates for (of sectors in input-output model consideration)</th>
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<tbody>
<tr>
<td></td>
<td>Total damage in £ for asset damage</td>
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<td>Loss of household customer demand in £</td>
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<td>Loss of government demand in £</td>
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<td>Loss of labour for infrastructures in £</td>
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<td>Export-import losses in £</td>
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- Demand estimates in British Pounds (£) for infrastructure assets resources
- Future scenarios of infrastructures from WS1

<table>
<thead>
<tr>
<th>EA flood maps and event probabilities</th>
<th>Intersection hazards and infrastructure assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPD flood extent maps</td>
<td>Determine asset fragility</td>
</tr>
<tr>
<td>JBA spatially coherent flood depths and probabilistic flood maps</td>
<td>Develop mechanisms in failure</td>
</tr>
<tr>
<td>CEH G2G flow estimates and return periods</td>
<td>Find directly damage assets and indirectly damaged assets from interdependency relationships</td>
</tr>
<tr>
<td>Wind, heat hazard model estimates</td>
<td>Find spatial extent of damage impacts due to asset failures</td>
</tr>
<tr>
<td>Topological capacity and flow networks for Great Britain</td>
<td>Find number of customers affected due to asset damage</td>
</tr>
<tr>
<td>Government Census population data for district/ward level with output areas</td>
<td>Recalibrate analysis through network capacity, flow and flux redistribution</td>
</tr>
<tr>
<td>Demand estimates in British Pounds (£) for infrastructure assets resources</td>
<td>Analysis for Great Britain</td>
</tr>
<tr>
<td>Capacity and demand and future scenarios of infrastructures from WS1</td>
<td>Number of assets failed due to direct hazard impact, indirect interdependency impact</td>
</tr>
<tr>
<td>Cost-adaptation analysis from WS5</td>
<td>Asset and network probabilities of failures</td>
</tr>
<tr>
<td></td>
<td>Spatial extent of asset damage as polygon area extents around assets</td>
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</tbody>
</table>
## UK Infrastructure Transitions Research Consortium

- Result sets for different times during analysis of asset and infrastructure network recovery

<table>
<thead>
<tr>
<th>Team/Agency</th>
<th>Phase</th>
<th>Input</th>
<th>Task/Tools</th>
<th>Output</th>
</tr>
</thead>
</table>
| Cambridge University (Peter Tyler, D.J. Crawford-Brown, Scott Kelly, Chris Thoung) | 1 | • Aggregated England level estimates for (of sectors in input-output model consideration)  
  o Total damage in £ for asset damage  
  o Loss of household customer demand in £  
  o Loss of business customer demand in £  
  o Loss of government demand in £ | • Compile national level input-output data for 87 economic sectors  
  • Generate supply side and demand side loss estimate model for macroeconomic analysis  
  • Perform analysis to estimate direct and indirect economic loss estimates for 87 sector economy | • Estimates for England level analysis for 87 economic sectors  
  o Economic output loss in £ for sectors  
  o Economic demand loss estimate in £ for sectors  
  o Direct and indirect economic in £ for sectors  
  o Amplification metrics (direct/indirect, etc.) for economic sectors  
  o Forward and backward linkage effects |
|  | 2 | • Aggregated England level estimates for (of sectors in input-output model consideration)  
  o Total damage in £ for asset damage  
  o Loss of household customer demand in £  
  o Loss of business customer demand in £  
  o Loss of government demand in £ | • Compile national level input-output data for 87 economic sectors  
  • Generate supply side and demand side loss estimate model for macroeconomic analysis  
  • Perform analysis to estimate direct and indirect economic loss estimates for 87 sector economy | • Estimates for Great Britain level analysis for 87 economic sectors  
  o Economic output loss in £ for sectors  
  o Economic demand loss estimate in £ for sectors  
  o Direct and indirect economic in £ for sectors  
  o Economic amplification metrics (direct/indirect, etc.) for economic sectors  
  o Forward and backward linkage effects |
| 3 | • Cost-adaptation analysis from WS5  
  • Aggregated Great Britain level estimates for (of sectors in input-output model consideration)  
    o Total damage in £ for asset damage  
    o Loss of household customer demand in £  
    o Loss of business customer demand in £  
    o Loss of government demand in £  
    o Loss of labour for infrastructures in £  
    o Export-import losses in £  
  • Result sets for different times during analysis of asset and infrastructure network recovery |
| 3 | • Compile national level input-output data for 87 economic sectors  
  • Generate supply side and demand side loss estimate model for macroeconomic analysis  
  • Perform analysis to estimate direct and indirect economic loss estimates for 87 sector economy  
  • Perform economic adaptation analysis for sector recoveries  
  • Estimates for Great Britain level analysis for 87 economic sectors  
    o Economic output loss in £ for sectors  
    o Economic demand loss estimate in £ for sectors  
    o Direct and indirect economic in £ for sectors  
    o Economic amplification metrics (direct/indirect, etc.) for economic sectors  
    o Forward and backward linkage effects  
  • Economic adaptation rates and temporal estimates of results |
6.3. Computation tools

Another flowchart, in Figure 10, shows the computational tools being used or proposed to be used in the implementation of the overall WS2 analysis. The flowchart also provides an overview of the information exchanges between different teams that develop models and tools for their analysis.

Figure 10: Flowchart showing computational tools and interactions between the ITRC WS2 teams in implementing the risk analysis framework.